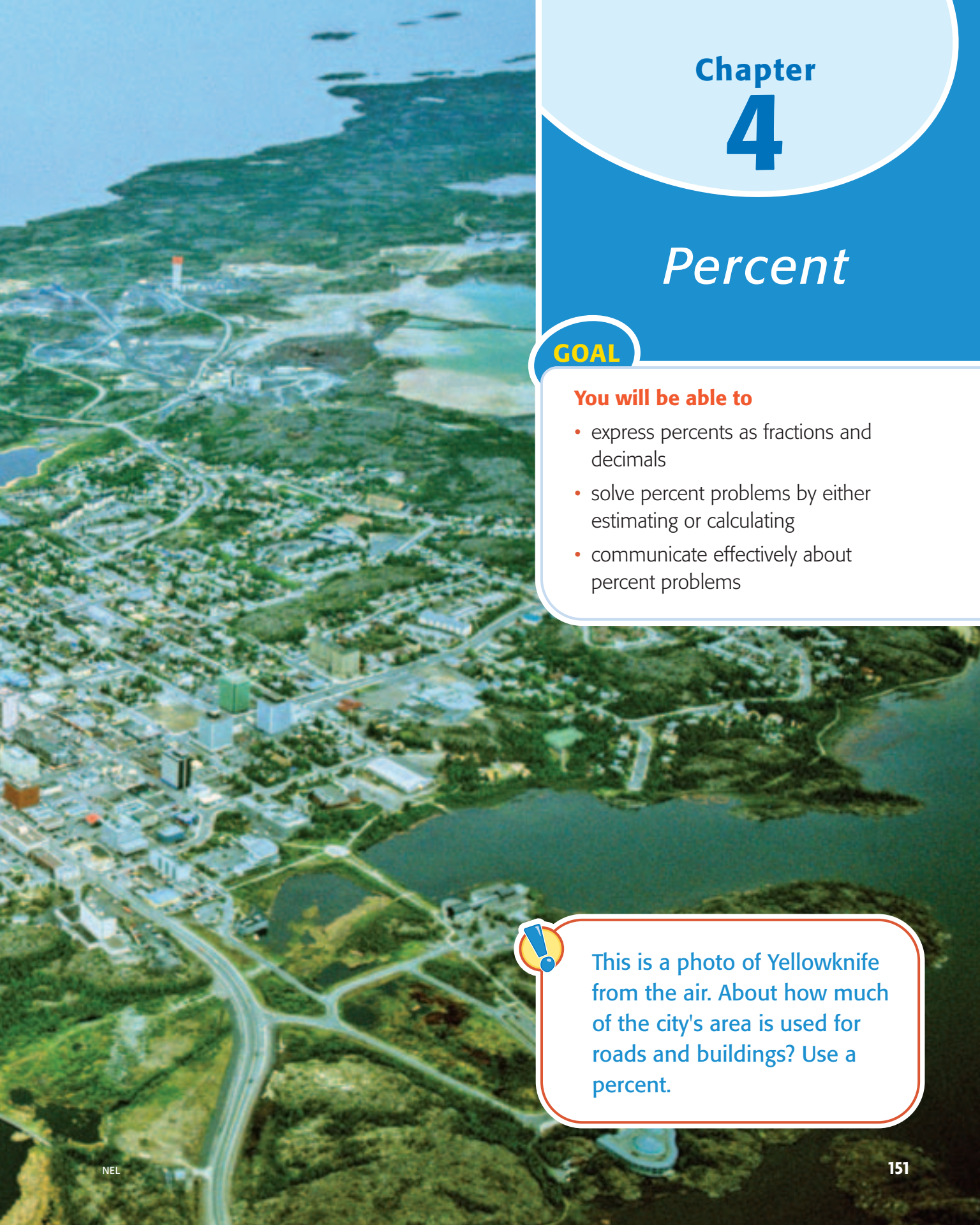# *Percent*

GOAL

You will be able to

- express percents as fractions and decimals
- solve percent problems by either estimating or calculating
- communicate effectively about percent problems

This is a photo of Yellowknife from the air. About how much of the city's area is used for roads and buildings? Use a percent.

Getting **Started**

Parts of a Whole

Some Canadian farmers build mazes in their cornfields to attract tourists. The following grid shows a plan for a simple maze in a square cornfield. The white squares are the paths.

Start

Finish
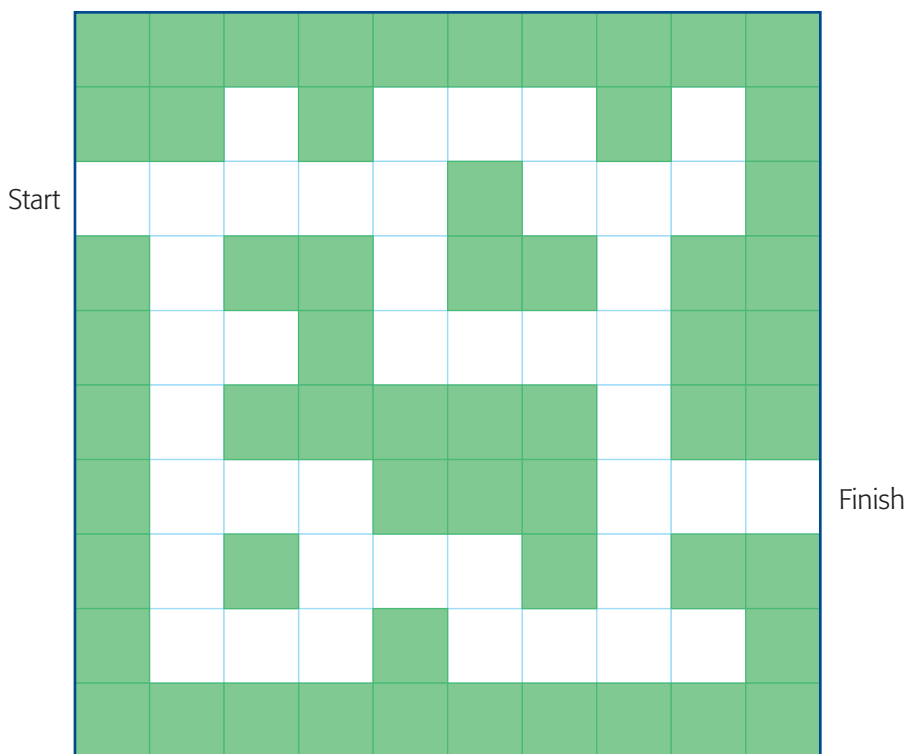
❓ How can you describe paths in the maze using fractions, decimals, and percents?

A. What fraction of the square cornfield is used for the paths in the maze? Write an **equivalent** decimal.

B. What fraction of the cornfield is used for the corn? Write an equivalent decimal.

C. What **percent** of the cornfield is used for the paths?

D. What percent of the cornfield is used for the corn?

E. Copy the maze onto grid paper, and trace a path from Start to Finish.

F. Count the squares on your path. Express the area of your path as part of the total area of the cornfield using fractions, decimals, and percents.

G. Explain why the areas of the paths in the maze are easily expressed as fractions, decimals, and percents of the cornfield's total area.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. If you get a score of 72% on a test, you must have had 72 questions right.

2. Your legs make up about 40% of your height.

3. 10% of an amount of money is not very much.

4. You can compare fractions, decimals, and percents on a number line.

4.1

Percents as Fractions and Decimals

GOAL

Solve percent problems using equivalent fractions or decimals.

LEARN ABOUT the Math

48% of Nestor's class stay for lunch. Nestor wants to represent this percent on a 5 × 5 grid. He knows that he can easily show 48% on a 10 × 10 grid.

❓ How many squares of the 5 × 5 grid should Nestor colour?

A. Write 48% as a fraction of 100 and as a decimal.

B. How could you rename your fraction in part A as an equivalent fraction to help you solve the problem?

C. How many of the 25 squares on Nestor's grid represent 48%?

Reflecting

D. How did renaming the percent help you solve the problem?

E. Why is it always easy to write a percent as a decimal?

WORK WITH *the Math*

Example 1 | Writing a percent as a fraction and a decimal

In Ashley's class, 75% of the students ride the bus to school. Write the number of students who ride the bus as a fraction in lowest terms and as a decimal.

Ashley's Solution

$$75\% = \frac{75}{100}$$

75% means 75 parts out of 100.

$$\overset{\div 25}{\frac{75}{100}} = \frac{3}{4}$$
$$\underset{\div 25}{}$$

To rename $\frac{75}{100}$ as a fraction in lowest terms, I divided the numerator and the denominator by the same number. I know that $\frac{3}{4}$ is in lowest terms because 3 and 4 have no common factors.

$$75\% = 0.75$$

I can rename $\frac{75}{100}$ as 0.75.

Example 2 | Expressing a fraction as a percent

According to a school newspaper, $\frac{2}{5}$ of the 750 students in the school signed a petition for less homework. What percent of students signed the petition?

Pavlo's Solution

$$\frac{2}{5} = \frac{4}{10}$$
$$\frac{4}{10} = \frac{40}{100}$$
40% of students signed the petition.

$$\frac{2}{5} < \frac{1}{2}, \text{ so } 40\% \text{ seems reasonable.}$$

To write $\frac{2}{5}$ as a percent, I needed to rename $\frac{2}{5}$ as an equivalent fraction with a denominator of 100.

I can rename $\frac{40}{100}$ as 40%.

I knew my answer must be less than 50%.

A Checking

1. Rename each percent as a fraction.

   a) $24\% = \dfrac{\blacksquare}{50}$

   b) $40\% = \dfrac{\blacksquare}{10}$

   c) $50\% = \dfrac{\blacksquare}{20}$

   d) $75\% = \dfrac{\blacksquare}{20}$

2. In Guilia's class, 36% of the students speak more than one language. Write the number of students who speak more than one language as a fraction in lowest terms.

B Practising

3. Rename each decimal as a fraction.

   a) $0.10 = \dfrac{\blacksquare}{10}$

   b) $0.34 = \dfrac{\blacksquare}{50}$

   c) $0.33 = \dfrac{\blacksquare}{100}$

   d) $0.2 = \dfrac{\blacksquare}{5}$

4. In Eric's class, 35% of the students have blond hair. Write the number of students with blond hair as a fraction in lowest terms.

5. Write each percent as a fraction in lowest terms.

   a) 22%   b) 5%   c) 30%   d) 72%

6. Write each percent as a decimal.

   a) 3%   b) 94%   c) 100%   d) 40%

7. Complete this table.

Percent	Decimal	Fraction in lowest terms
60%		
	0.09	
		$\dfrac{3}{100}$
44%		
		$\dfrac{6}{25}$
	0.5	
		$\dfrac{3}{3}$
12%		

8. Match each percent to the figure that represents it.

a) 50%   **b)** 75%   **c)** 25%   **d)** 40%

A.   **B.**   **C.**   **D.**

9. Claudia's new coat is made of 60% wool, 30% polyester, and 10% nylon. Write each percent as a fraction in lowest terms.

10. Complete each statement using $<$, $>$, or $=$. Explain your answers.

a) $0.3 \ \blacksquare \ 30\%$

b) $0.45 \ \blacksquare \ \dfrac{7}{45}$

c) $\dfrac{3}{5} \ \blacksquare \ 40\%$

d) $20\% \ \blacksquare \ \dfrac{4}{25}$

e) $\dfrac{2}{8} \ \blacksquare \ 25\%$

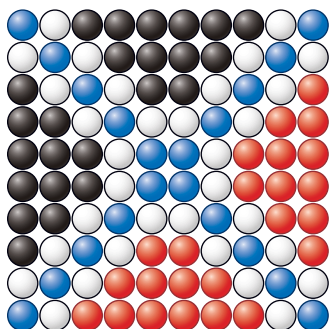**f)** $42\% \ \blacksquare \ 4.2$

11. What fraction with a numerator of 1 or 2 could you use to estimate each percent? Explain.

a) 30%   **b)** 15%   **c)** 70%   **d)** 9%

12. a) Describe each colour of beads as a percent of the entire design.

b) Write each percent as a fraction in lowest terms.

13. The air you breathe is 20 parts oxygen and 80 parts other gases.

a) What percent of the air is oxygen? What percent of the air is made up of other gases?

b) Write each percent as a fraction in lowest terms.

14. At a provincial campground, 25% of the sites are for tent camping, 60% are for RVs, and the rest are for groups.

a) What percent of the sites are for groups?

b) The campground has 152 sites. How many sites are for tent camping?

15. How do you know, without calculating, that the percent for $\dfrac{7}{25}$ is less than the percent for $\dfrac{7}{20}$?

4.2

Investigating Percents

GOAL

Explore how percents can be used.

EXPLORE the Math

Nutrition Facts
Valeur nutritive
Per 1 cup (50 g) / pour 1 tasse (50 g)

Amount Teneur	% Daily Value % valeur quotidienne
Calories / Calories 190	
Fat / Lipides 0 g	0 %
Saturated / saturés 0 g + Trans / trans 0 g	0 %
Cholesterol / Cholestérol 0 mg	
Sodium / Sodium 290 mg	12 %
Carbohydrate / Glucides 44 g	15 %
Fibre / Fibres 0 g	0 %
Sugars / Sucres 2 g	
Protein / Protéines 3 g	
Vitamin A / Vitamine A	0 %
Vitamin C / Vitamine C	0 %
Calcium / Calcium	0 %
Iron / Fer	2 %

In 2001, Health Canada recommended that food packages should have a Nutrition Facts label. This label should include the size of one serving, the amounts of the nutritional components (such as calories, fats, carbohydrates, sodium, and some vitamins), and the percent of the recommended daily intake for each component.

Find a Nutrition Facts label from a snack food. Share the label with a group of classmates.

According to the Nutrition Facts labels that your group found, which is the best snack?

4.3 Estimating Percents

Develop and apply estimation strategies for percents.

LEARN ABOUT the Math

To prepare for the upcoming baseball season, Nick goes shopping for a new baseball glove and helmet. A sporting goods store is advertising a sale on baseball equipment.

? About how much will the baseball equipment cost?

Example 1 | Estimating percent using fractions

A $40 baseball glove is 35% off. About how much does it cost, without tax?

Nick's Solution

The cost is the original price minus the amount of the discount.	I didn't calculate an exact amount, because I only needed to know about how much the glove costs.
$35\% = \dfrac{35}{100}$	I wanted to estimate 35% with an easy fraction.
$\dfrac{35}{100}$ is about $\dfrac{1}{3}$.	$100 \div 35$ is close to 3, so I estimated with $\dfrac{1}{3}$.
$40 - 13 = 27$ The glove costs about $27.	I estimated by subtracting $\dfrac{1}{3}$ of 39 from 40. I used 39 instead of 40 because 39 is a multiple of 3.

Example 2 | Estimating percent using decimals

After a discount, a batting helmet sells for $18.84, plus 5% tax. Is $20 enough to buy the helmet?

Jessica's Solution

The difference between $18.84 and $20.00 is about $1.15. If the tax is less than $1.15, then the purchase price is less than $20.

I estimated because I just needed to know whether $20 is enough, not the exact purchase price.
I used easy numbers to estimate the tax.

10% of $18.84 = $1.88

10% is a good benchmark because it's easy to calculate. To calculate 10%, I moved the digits one place to the right.

Half of $1.88 is less than $1, so 5% of $18.84 is less than $1.
$1 is less than $1.15, so $20 is enough to buy the helmet.

5% is half of 10%.

Gail's Solution

$18.84 is about $19.00.

I rounded the cost of the helmet up to the nearest dollar.

1% = 0.01

1% is a good benchmark because it's easy to calculate, and 5% is 5 × 1%.

0.01 x 19.00 = 0.19

To determine 1% of $19.00, I multiplied by 0.01.

19 x 5 = 20 x 5 − 5, which is 95. So, 0.19 x 5 is 0.95.

Then I multiplied by 5.

19.00 + 0.95 = 19.95
$20 is enough to buy the helmet.

I added the cost and the tax to get the purchase price.

Reflecting

A. Why did Nick try to use a fraction with a numerator of 1 to solve the problem?

B. Why were Jessica's and Gail's approaches to solving the problem useful?
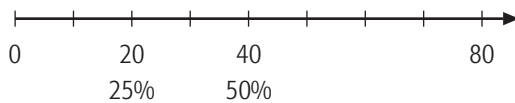
WORK WITH the Math

Example 3	Estimating a percent using a number line

In last year's softball games, Ashley had 34 hits in 80 times at bat. About what percent of Ashley's times at bat did she get a hit?
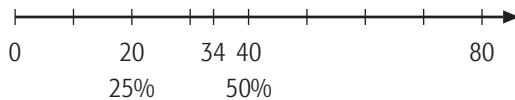
Solution

Use a number line to estimate $\frac{34}{80}$ as a percent.

Choose useful benchmarks, such as 20 and 40.

34 is about $\frac{3}{4}$ of the way from 20 to 40.

$50 - 25 = 25$

$\frac{1}{4}$ of 25 is about 6, so

$\frac{3}{4}$ of 25 is about 3 x 6 = 18

The equivalent percent should also be about $\frac{3}{4}$ of the way from 25% to 50%.

$25\% + 18\% = 43\%$

Ashley had a hit in about 43% of her times at bat.

Communication | *Tip*

Sometimes writing out what the numerator and the denominator represent in a fraction can be helpful. For example, for 34 hits in 80 times at bat, write

$$\frac{\text{number of hits}}{\text{number of times at bat}} = \frac{34}{80}.$$

A Checking

1. What benchmark would you use to estimate what percent 8 out of 18 is?

2. **a)** Estimate 24% of 42. **b)** Estimate 65% of 29.

B Practising

3. Estimate what percent 9 is of 38. Explain your answer.

4. Estimate each percent. Explain what benchmark you used to make your estimate.
   a) 9% of 27   **c)** 76% of 25
   b) 32% of 62   **d)** 89% of 48

5. Are your estimates in the previous question greater or less than the actual answers? Explain.

6. Estimate what percent each fraction is. Explain what benchmark you used to make your estimates.
   a) $\dfrac{12}{25}$ **b)** $\dfrac{22}{80}$ **c)** $\dfrac{67}{300}$ **d)** $\dfrac{31}{148}$

7. Are your estimates in the previous question greater or less than the actual answers? Explain.

8. Some of the students in Alan's school want to have a winter carnival on a school day. The principal will allow them to have the carnival if 90% of the students in the school sign up. So far, 448 of the 550 students in the school have signed up. Estimate to determine whether enough students have signed up. Explain what you did.

9. During the first hour of a powwow, 47 people arrived. The other 85% arrived later in the day. About how many people attended the powwow?

10. You need 60% on your test to get a higher mark in a subject. You got 19 out of 26 marks. Why can you estimate to decide if you have the 60% you need?

11. **a)** Name a situation in which you might estimate a percent.
    b) When would you need to calculate a percent exactly?

4.4 Using Percents to Make Comparisons

Compare fractions and ratios using percents.

LEARN ABOUT the Math

Pavlo wrote a science test and a social studies test this week. He scored 21 out of 30 on the science test and 18 out of 25 on the social studies test. He is trying to determine which test he did better on.

On which test did Pavlo score a higher mark?

A. Write fractions that describe Pavlo's marks on the tests.

B. Can you tell from the fractions which is the higher mark? Explain your answer.

C. Rewrite the fractions as equivalent fractions with the same denominator.

D. Write each test mark as a percent.

E. On which test did Pavlo score a higher mark?

Reflecting

F. Why were the fractions in part C easier to compare than the fractions in part B?

G. Why is writing two fractions as percents like writing them as equivalent fractions?

WORK WITH the Math

Example 1 | Comparing different-sized groups

One year in Vancouver, 68% of the days in January and 68% of the days in February had some precipitation. Did one month have more days with some precipitation? (This was a non-leap year.)

Jessica's Solution

68% of 31 = 0.68 x 31 = 21.08
That's 21 days.
68% of 28 = 0.68 x 28 = 19.04
That's 19 days.

68% of 31 is greater than 68% of 28. January had more days with some precipitation.

There are 31 days in January. In a non-leap year, there are 28 days in February. I had to compare 68% of 31 and 68% of 28. 68% of a greater amount should be more. If the numbers are too close, however, there may be no difference.

31 and 28 were far enough apart for 68% of 31 and 68% of 28 to be different whole numbers.

Example 2 | Comparing ratios

On the first day of the Heritage Fair, the ratio of children to adults was 2:5. On the second day, the ratio was 3:7. On which day did a greater percent of children attend the fair?

Nestor's Solution

On the first day, $\frac{2}{7}$ of the people were children. On the second day, $\frac{3}{10}$ of the people were children. I knew that $\frac{3}{10}$ is 30%.

2 ÷ 7 = 028571

$\frac{2}{7}$ is about 29%.

A greater percent of children attended the Heritage Fair on the second day.

The ratio 2:5 means that for every 7 people, 2 were children and 5 were adults. The ratio 3:7 means that for every 10 people, 3 were children and 7 were adults.

I rewrote $\frac{2}{7}$ as a percent using my calculator.

A Checking

1. Jerry's lacrosse team won 10 of 25 games. Joel's basketball team won 18 of 45 games.
   a) What percent of the games did each team win?
   b) Who had the better record for wins?

2. Caroline is doing a traffic survey. On the first day, the ratio of cars with passengers to cars with only a driver was 13:7. On the second day, the ratio was 8:5. On which day was the percent of cars with passengers greater? How do you know?

B Practising

3. Tom mixes a punch with 4 parts ginger ale to 6 parts fruit juice. Lina mixes a punch with 3 parts ginger ale to 5 parts fruit juice. Whose punch has a greater percent of ginger ale?

4. On Monday at the Pacific National Exhibition, there were 4 children for every 3 adults. On Tuesday, there were 6 children for every 4 adults. On which day did a greater percent of children attend?

5. Order the fractions $\frac{23}{25}$, $\frac{28}{30}$, $\frac{47}{50}$, and $\frac{37}{40}$ from greatest to least.

6. After taxes, Tara paid $27 for a $25 purchase, and Jolene paid $32.10 for a $30 purchase.
   a) How much did each girl pay in taxes?
   b) What fraction of each original price were the taxes?
   c) Who paid the higher percent of taxes?

7. According to a weather forecast, there is a 30% chance of rain. Explain what you think this means.

8. Why is 100% of a week not as long as 10% of a year?

9. Sandeep deposited $500 in a bank account. After one year, the amount increased to $525. Sunil deposited $400 in a bank account. After one year, the amount increased to $420. Who had the better interest rate? Explain.

10. Describe two different ways to compare $\frac{3}{5}$ and $\frac{4}{6}$. One way must involve percents.

Mid-Chapter **Review**

Frequently Asked Questions

Q: **How are percents, fractions, and decimals related?**

A: They are all ways to describe ratios or parts of amounts. If you know one form (percent, fraction, or decimal), you can determine the other two equivalent forms.

For example, 80% means 80 out of 100 or $\frac{80}{100}$, which is equivalent to $\frac{4}{5}$. The fraction $\frac{80}{100}$ means 80 hundredths, which is equivalent to 0.80 or 0.8.

Q: **How do you estimate a percent?**

A: Use benchmarks (numbers that are easy to use) to estimate. For example, 24% of 38 is about 25% of 40. Since 25% of 40 is $\frac{1}{4}$ of 40, 24% of 38 is about 10.

Practice

Lesson 4.1

1. How many cents does each percent of a dollar represent?
  a) 1%   **b)** 72%   **c)** 100%   **d)** 40%

2. Complete each statement using <, >, or =.
  a) 15% ▦ 0.2   **c)** 0.8 ▦ 75%
  b) $\frac{1}{4}$ ▦ 25%   **d)** 0.5 ▦ 5%

3. There are 16 children and 24 adults at an art show.
  a) What percent of the people are children?
  b) What percent of the people are adults?

4. Estimate each percent. Show your work.

a) 42 out of 52

c) $19.99 compared to $79.97

b) $\dfrac{18}{47}$

d) 29 seconds out of 1 minute

5. Is each statement reasonable or unreasonable? Explain how you know.

a) 12% of 42 is about 4.

c) 72% of 60 is about 58.

b) 35% of 150 is about 50.

d) 45% of 180 is about 90.

6. Which benchmarks would you use to estimate each amount? Estimate each amount using these benchmarks.

a) a 15% tip on a restaurant bill of $39.45

b) 6% tax on an item costing $41.95

7. Bones account for about 20% of a person's total mass. Estimate the mass of the bones in a 62 kg person.

8. Barry got 17 out of 20 on a science test and 39 out of 50 on a math test. On which test did he do better? Explain.

9. Tracy and Tristan both earn a percent of any sales they make at a furniture store. Last month, Tracy earned $750 on sales of $5000, and Tristan earned $825 on sales of $6875. Who earns the greater percent? Show how you know.

10. Explain which sale offers the greatest discount.

Discount Phone Cards
You pay just $15 for a $20 card

PHONE CARDS SALE!!
Were $10
Now only $7

$10
Save on phone cards
$15

4.5

Calculating with Percents

GOAL

Solve problems that involve percents using equivalent ratios.

LEARN ABOUT the Math

Nick and his family went out to dinner to celebrate his graduation. Their bill came to $53.00. The family usually leaves a 15% tip for the server.

❓ **How much tip should Nick's family leave for the server?**

Example 1 | Calculating the percent of a total

Calculate the tip that Nick's family should leave.

Nick's Solution

proportion

a number sentence that shows two equivalent ratios or fractions; for example, $\frac{1}{4} = \frac{5}{20}$

I rounded $53 to $50.
I used the **proportion** $\frac{15}{100} = \frac{\blacksquare}{50}$ to estimate the tip.

We don't need an exact amount. I needed the number that is 15% of 50. I wrote this number as $\blacksquare$. I knew that $\frac{\blacksquare}{50}$ is equivalent to $\frac{15}{100}$, because they are both 15%.

$$\frac{15}{100} \overset{\div 2}{\underset{\div 2}{=}} \frac{7.5}{50}$$

Since I divided 100 by 2 to get 50, I had to divide 15 by 2 to get an equivalent fraction.

We should leave about $7.50 for the tip.

Gail's Solution

$50 \div 10 = 5$
$5 \div 2 = 2.5$
$5 + 2.5 = 7.5$
15% of $50 is $7.50

Nick's family should leave $8 for the tip.

I estimated $53 as $50. I calculated 10% of 50 by dividing by 10.
I calculated 5% as half of 10%.
I added 10% and 5% to get 15%.

I added a little bit extra because $53 is greater than $50.

Reflecting

A. Why was it appropriate to estimate 53 with 50?

B. Why is a 15% tip easier to calculate than a 13% tip?

WORK WITH the Math

Example 2 | Calculating a percent

A group of 20 Grade 7 students were surveyed about their favourite type of music. This table shows the results of the survey. What percent of the students did not choose rap?

Type of music	Number of students
country	1
rock	4
hip-hop	6
rap	9

Solution A: Using a grid

Draw a grid of 20 squares, since 20 students were surveyed. Make the grid 2 squares wide by 10 squares high.
Shade 11 squares for students who chose country, rock, or hip-hop. There are 9 unshaded squares for students who chose rap.
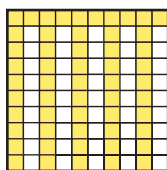
Copy the grid four more times to make 100 squares.
Each group of 20 has 11 shaded squares. Multiply 11 by 5 to get the number of shaded squares in the 100 grid.

$5 \times 11 = 55$

$\frac{55}{100} = 55\%$, so 55% of the students did not choose rap.

Solution B: Using a proportion

$\frac{\text{number who did not choose rap}}{\text{number of students}} = \frac{11}{20}$

11 students did not choose rap.
Write the ratio as a fraction.

$\frac{11}{20} = \frac{\blacksquare}{100}$

Write a proportion. Since you're calculating a percent, put the missing term over 100.

$\frac{11}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{\blacksquare}{100}$

$20 \times 5 = 100$, so $11 \times 5 = \blacksquare$.

$\frac{11}{20} = \frac{55}{100}$

$11 \times 5 = 55$

55% of the students did not choose rap.

Example 3 | **Calculating an amount from a percent**

Suppose that Nick's family had spent $80 for his graduation dinner.
How much would a 15% tip be?

Solution

$$\frac{15}{100} = \frac{tip}{80}$$

15% means "15 out of 100," so the tip must be the same portion of 80.

$$\overset{\div 5}{\frac{15}{100}} = \underset{\div 5}{\frac{3}{20}}$$

First rewrite $\frac{15}{100}$ as an equivalent fraction in lower terms.

$$\frac{3}{20} = \frac{tip}{80}$$

Then rewrite $\frac{3}{20}$ as an equivalent fraction with a denominator of 80.

$$\overset{\times 4}{\frac{3}{20}} = \underset{\times 4}{\frac{12}{80}}$$

A 15% tip for an $80 dinner is $12.

Example 4 | **Calculating a number from a percent**

There are 10 boys in Jessica's music class. According to the teacher, 40% of the students in the class are boys. How many students are in the class?

Solution

$$\frac{40}{100} = \frac{10 \text{ boys}}{\text{total number of students}}$$

40% means 40 boys out of 100. Write the ratio as a fraction.
Write a proportion. The missing term in the proportion is the total number of students.

$$\overset{\div 4}{\frac{40}{100}} = \underset{\div 4}{\frac{10}{\text{total}}}$$

40 ÷ 4 equals 10, so the total number of students must be 100 ÷ 4.

total = 100 ÷ 4 = 25
There are 25 students in the music class.

Divide.

A Checking

1. Calculate 15% of 60 using each method.
   a) Use mental math:
      - Think of 10% of 60.
      - Take half of your number above.
      - Add the two numbers.
   b) Use a proportion:
      - Rewrite $\dfrac{15}{100}$ in lowest terms.
      - Multiply to rename this fraction as an amount out of 60.

2. Determine each missing number.
   a) 20% of ▇ = 45   b) 12% of ▇ = 54

3. Estimate the 15% tip for each dinner bill.
   a) $26.50   b) $13.22

B Practising

4. Calculate.
   a) 50% of 20   d) 12% of 50
   b) 75% of 24   e) 15% of 200
   c) 20% of 45   f) 44% of 250

5. Calculate.
   a) 50% of ▇ = 15   d) 75% of ▇ = 12
   b) 25% of ▇ = 22   e) 15% of ▇ = 24
   c) 10% of ▇ = 7   f) 44% of ▇ = 55

6. Out of 600 computers produced in a factory, 30 failed to pass inspection due to bad disk drives. What percent failed to pass inspection?

7. There are 12 girls with blond hair in Katya's gymnastics class. This is 25% of the entire class. Using mental math, calculate the total number of students in the class.

8. About 14% of Canadians who are at least 100 years old live in British Columbia. On the 2001 census, 531 people in British Columbia said they were at least 100 years old. About how many Canadians were at least 100 years old in 2001?

9. A dealer paid $6000 for a used car. The dealer wants to make a profit that is 25% of the price he paid for the car.
   a) What profit does the dealer want to make?
   b) For how much should the dealer sell the car?

10. If Sarah's mother really likes the service in a restaurant, she leaves a 20% tip. The family's last bill was $110, and Sarah's mother left $22 for the tip. Did she really like the service? Explain.

11. There are four common blood types: O, A, B, and AB. The diagram below shows the percent of people with each blood type in a specific group. In a group of 2500 people, how many would you expect to have each blood type?

O: 45%
A: 41%
B: 10%
AB: 4%
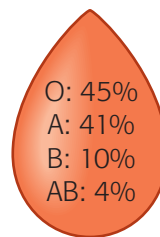
12. A new process in a factory has increased production by 12%. Workers are now producing 30 more skateboards each day. How many skateboards did they produce each day before the new process was introduced?

13. Explain how you could use a proportion to express $\frac{165}{250}$ as a percent.

4.6

Solving Problems that Involve Decimals

**GOAL**

Solve problems that involve percents and decimals.

LEARN ABOUT the Math

In the student council election, Gail received 168 out of 240 votes.

What percent of the votes did Gail receive?

A. Write the ratio of votes Gail received to total votes as a fraction.

B. Write a proportion you could solve to determine a fraction with a denominator of 100 that is equivalent to your fraction in part A.

C. What percent of the votes did Gail receive?
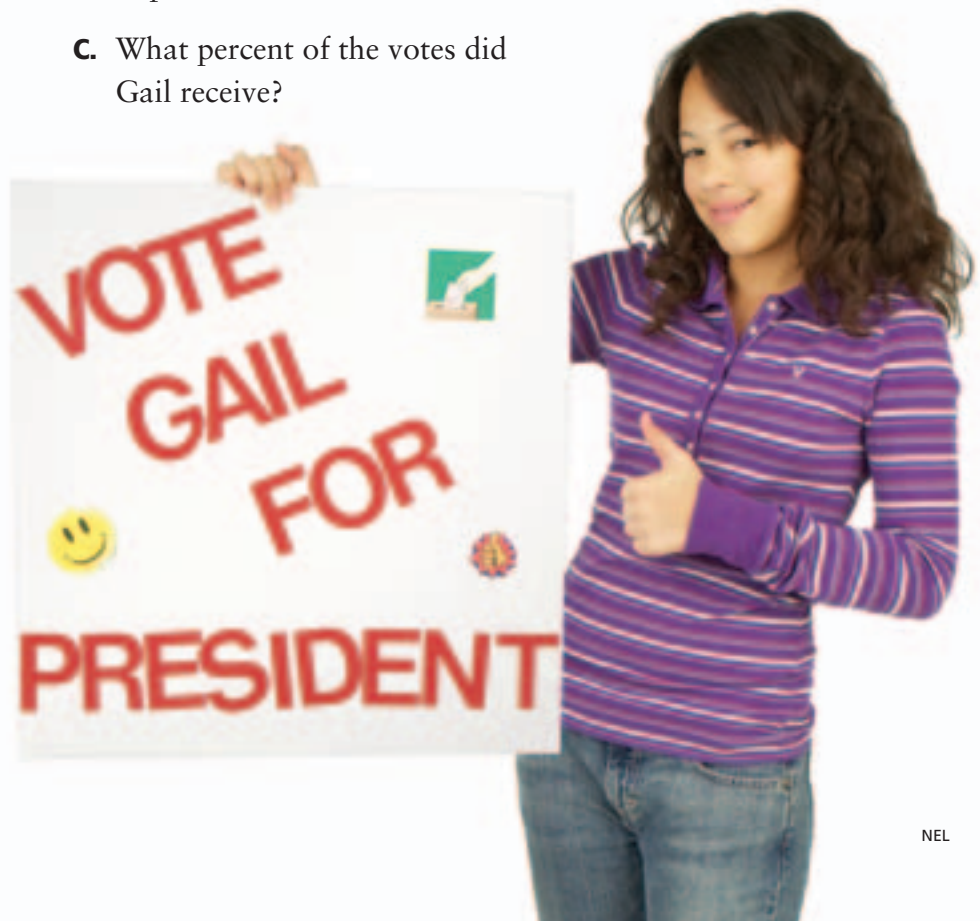
Reflecting

D. Why was it helpful to use an equivalent fraction with a denominator of 100 to calculate the percent?

E. Why would the problem have been harder to solve for an exact percent if Gail had received 170 out of 240 votes?

WORK WITH *the Math*

Example 1 | Calculating a percent

In the student council election, Pavlo received 72 out of 120 votes. Determine the percent of the votes he received.

Pavlo's Solution

$\dfrac{\text{votes for Pavlo}}{\text{total votes}} = \dfrac{72}{120}$

I wrote the first ratio, which is the number of votes I received compared to the total number of votes.

$\dfrac{72}{120} = \dfrac{\blacksquare}{100}$

I used ■ to represent the number of votes I received out of every 100 votes.

$120 \div 100 = 1.2$

I divided 120 by 100 to determine the number I should divide 72 by.

$\dfrac{72}{100} \overset{\div 1.2}{\underset{\div 1.2}{=}} \dfrac{\blacksquare}{100}$

I divided 72 by 1.2 to get the percent.

$72 \;\boxed{\div}\; 1.2 \;\boxed{=}\; \boxed{\quad 60.\quad}$

$\dfrac{72}{100} \overset{\div 1.2}{\underset{\div 1.2}{=}} \dfrac{60}{100}$

When I divided the numerator and the denominator by 1.2, I got an equivalent fraction with a denominator of 100.

$\dfrac{60}{100} = 60\%$

I wrote the fraction as a percent.

I received 60% of the votes.

Example 2 | Comparing amounts

In 1989, close to 42 million litres of oil were spilled in Prince William Sound, Alaska. Cleanup crews were able to recover 14% of the spilled oil.

In 2005, about 32 million litres of oil were spilled because of Hurricane Katrina. Cleanup crews recovered about 22.4 million litres of this oil.

Compare the amounts of oil recovered after the two disasters.

Nestor's Solution

I used percents to compare the oil recovered after the two disasters.

$$\frac{22.4}{32} = \frac{\blacksquare}{100} \quad \times ?$$

I knew the percent of oil recovered in Prince William Sound. I needed to calculate the percent recovered after Hurricane Katrina.

I had to determine what percent 22.4 is of 32, so I wrote a proportion.

I needed a number that I could multiply 32 by to get 100.

$32 \times \blacksquare = 100,$
so $100 \div 32 = \blacksquare$.

I couldn't solve this, but I was able to write the multiplication as a division.

$100 \div 32 =$

I used my calculator.

$$3.125$$

$$\frac{22.4}{32} = \frac{70}{100} \quad \times 3.125$$

When I multiplied the numerator and the denominator by 3.125, I got an equivalent fraction with a denominator of 100.

$$\frac{70}{100} = 70\%$$

70% of the oil was recovered after Hurricane Katrina. This is five times greater than the percent of oil recovered in Prince William Sound.

I wrote this fraction as a percent.

Pavlo's Solution

I compared the quantities of oil recovered after the two disasters.

0.14 ⊠ 32 ⊟ 4.48

> 4.48

22.4 ⊡ 4.48 ⊟ 5.00

> 5.00

Five times the percent of oil was recovered after Hurricane Katrina as after the Prince William Sound spill.

I knew that 14% of the oil was recovered in Prince William Sound. I calculated what 14% of the Hurricane Katrina spill would be. The decimal equivalent of 14% is 0.14, so I calculated 14% of 32 by multiplying 0.14×32 on my calculator.

I compared the amount actually recovered to the amount I calculated.

A Checking

1. Calculate each percent.
   a) 7% of $49.98
   b) 10% of $27.99
   c) 9% of $69.86
   d) 8% of 3000

B Practising

2. Calculate 6% of each price.
   a)

   $14.95

   b)

   $59.95

3. Calculate the total price of each item, including 6% tax.

a)

$18.90

c)

$1.19

b)

$48.99

d)

$279.99
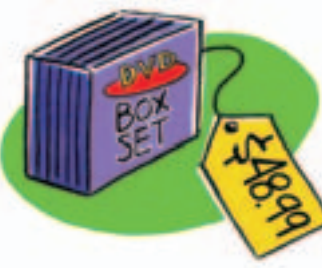
Province	PST
BC	7%
AB	0%
SK	5%
MB	7%

4. A tractor costs $39 500. How much would a farmer in Manitoba pay for the tractor after GST and PST?

5. Richard works at a clothing store. He is paid $150 a week, plus 9% of the value of his sales for the week. Last week, his sales totalled $457.85. Calculate his total earnings for the week.

6. Marie received a 4% raise to her $275 per week salary. She also earned 3% of the value of her sales, which were $1500 that week. How much money did she earn that week?

7. Jorge earned $30 when he collected $375 for newspaper subscriptions. What percent of the total did he earn?

8. A store is offering a 10% early-bird discount on a watch that regularly costs $24.00.
   a) What is the sale price of the watch?
   b) Calculate the total cost of the watch in British Columbia.

9. Choose a page of advertisements in a newspaper.
   a) Calculate the total cost of any two items on the page at 6%, 7%, and 8% sales tax.
   b) Why is the difference between taxes at 6% and 7% greater for items that cost more?

10. Give an example that shows how writing a proportion can help you solve a problem that involves percents.

Elastic Percents

You don't need a calculator to calculate percents if you have a piece of elastic.

Stretch the elastic along the metre stick. Make a mark every 1 cm along the elastic so that each mark represents 1%. Label each 5 cm.

1. To determine 60% of 75, place the 0% mark on the elastic at the 0 cm mark on the metre stick. Stretch the other end of the elastic until it reaches 75 cm. What number on the metre stick aligns with the 60% mark on the elastic?

2. Determine each amount with your percent elastic.
   a) 25% of 84   c) 80% of 115
   b) 35% of 60   d) 50% of 72

3. Estimate each amount with your percent elastic.
   a) 18% of 55   c) 70% of 69
   b) 48% of 120   d) 12% of 65

4. Why does your percent elastic work?

4.7 Solve Problems Using Logical Reasoning

GOAL

Solve problems that involve percents using logical reasoning.

LEARN ABOUT the Math

On a school ski trip, 10 students sign up for snowboarding. This is 40% of the students going on the trip.

? How many students are going on the school ski trip?

Example 1 | Using logical reasoning

Determine the number of students going on the trip.

Nick's Solution

Percent of group not snowboarding

= 100% − 40%

= 60%

Number of students not snowboarding

= 60% of group

I knew that 40% of the group is snowboarding, so 60% of the group is not snowboarding.

= 40% of group + 20% of group

= 10 + 5

= 15

40% of the group is equivalent to 10 students. Since 20% is half of 40%, 20% of the group is half of 10, which is 5.

Total number of students

= 10 + 15

= 25

15 students in the group are not snowboarding.

There are 25 students going on the school ski trip.

Reflecting

A. Why did Nick use 40% + 20% to calculate 60%?

B. Could Nick have solved this problem without determining the number of students who are not snowboarding?

WORK WITH the Math

Example 2 | Using logical reasoning and proportions

The African bush elephant weighs 6 t. This is about 4% of the mass of a blue whale. What is the mass of a blue whale?

Solution A: Comparing equivalent fractions

4% = 6 t	6 t of the blue whale's mass is equivalent to 4%.
25 × 4% = 25 × 6 t 100% = 150 t	100% is 25 times as much as 4%, so the missing amount must be 25 times as big as 6. Multiply both sides of the equation by 25.

The mass of a blue whale is 150 t.

Solution B: Comparing numerators of equivalent fractions

4% = 6 t	6 t of the blue whale's mass is equivalent to 4%.

8% = 12 t
2% = 3 t
10% = 8% + 2%
    = 12 t + 3 t
    = 15 t
	8% is twice as much as 4%.
	2% is half as much as 4%.

100% = 10 × 15 t = 150 t	10% × 10 = 100%

The mass of a blue whale is 150 t.

Example 3 | Comparing percents

About 17% of the students in Ms. Gregg's Grade 7 class wear glasses. About 19% of the students in Mr. Singh's Grade 7 class wear glasses. Can you tell which class has more students who wear glasses?

Solution

Since you don't know how many students are in each class, you can't tell which class has more students who wear glasses.

A Checking

1. On a class field trip, 15 students have sandwiches for lunch. This is 60% of the students on the trip. How many students went on the trip?

B Practising

2. How much would you save if there is a 20% discount on a bike that sells for $259.99?

3. In Canada, about 10% of the people are left-handed. In Fiji, about 40% of the people are left-handed.
   a) In a group of 30 Canadians, how many would you expect to be left-handed?
   b) In a group of 30 Fijians, how many would you expect to be left-handed?

4. A cheetah can run as fast as 112 kilometres each hour. A leatherback turtle can swim at about 30% of this speed. How fast can the turtle swim?

5. For every nine students in a school, two signed up for intramural sports. That was 208 students. One third of the students in the school are in Grade 7. How many students are not in Grade 7?

6. Estimate which is the better deal on a computer. Explain your reasoning.

7. When water freezes, its volume increases by about 10%.
   a) What is the increase in volume if 150 L of water freezes?
   b) What is the new volume?
   c) What is the new volume when 22.5 L of water freezes?

8. The lifespan of a black bear is about 40 years. The lifespan of a wolf is about 45% of the lifespan of a black bear. What is the lifespan of a wolf?

9. A set of number patterns is formed by starting with a number and then doubling it to get the next number. However, if the next number is greater than 20, subtract 15.
   For example, the pattern could be 3, 6, 12, 9, ...
   (since $2 \times 12 = 24$, and then $24 - 15 = 9$).
   a) What would the next number be in the pattern that starts with 3?
   b) If the 4th number in a pattern is 11, what could the start number be?

10. Hippopotamuses spend much of their time in water. You can hold your breath about 40% as long as a hippopotamus.
    a) How long can you hold your breath?
    b) How long do you think a hippopotamus can hold its breath?

11. A pair of jeans usually costs $80.00. The jeans are on sale at Jane's Jean Shop for 50% off. Denim Discounters offers the same jeans at 30% off, with a further 20% off the discounted price. Are the jeans the same price at both stores? Justify your answer.

12. Budget cuts caused the wages of employees at a shirt factory to be reduced by 5%. Profits at the end of the year were better than expected, so employees were given a 5% raise on their current wages. Are the employees' wages back to what they were before the salary cut? Justify your answer.

13. In a survey of 100 students who brought lunches,
    • 38 students brought fruit
    • 82 students brought a sandwich
    • 34 students brought both fruit and a sandwich
    a) How many students brought neither fruit nor a sandwich?
    b) Explain your strategy for solving the problem in part (a).

Recycling Bin Basketball

Tape a line on the floor, 3 m from your basket. Create a table like the one shown to record your team's results.

Number of players: 4 per team

How to Play

1. Each player on your team takes as many shots as she or he wants, up to a maximum of 10. Record each player's "Shots in basket" and "Shots taken" as a ratio in a table.

2. Calculate the "Percent in basket" for each player on your team. Round the percent to the nearest whole number, and record it in your table.

3. Calculate your team's "Percent in basket" for the total shots taken. Round to the nearest whole number, if necessary.

4. Compare your team's results with the results for the other teams in your class. The team with the highest "Percent in basket" wins.

Nick's Turn

I missed my first shot, but got 3 of the next 4 shots in the basket. I'll stop after 5 shots. Our team scored 12 out of 25 shots. That's 48% in the basket.

Player	Ratio of $\frac{\text{shots in basket}}{\text{shots taken}}$	Percent in basket
Gail	$\frac{4}{10}$	40%
Nestor	$\frac{2}{7}$	29%
Jessica	$\frac{3}{3}$	100%
Nick	$\frac{3}{5}$	60%
Total	$\frac{12}{25}$	48%

Percent	82%		
Fraction in lowest terms		$\frac{7}{20}$	
Decimal			0.07

1. Complete the table at the left.

2. Calculate.
   a) 1% of 600
       3% of 600
   b) 10% of 40
       90% of 40

3. Complete each statement.
   a) 40% of 35 is ■.
   b) ■% of 30 is 21.
   c) 60% of ■ is 39.
   d) 4.2 is ■% of 21.

4. A baseball stadium is 65% full. The capacity of the stadium is 1800. About how many people are in the stadium?

5. Akeem bought a video game for $38. He sold it a few days later for 40% less than he paid. How much did he sell the video game for?

6. Why is calculating 10% of an amount easier than calculating 17% of the amount?

7. a) Express each of Sue's test marks as a percent.
   b) Order her tests from lowest mark to highest mark.

$\frac{17}{25}$ $\frac{21}{30}$ $\frac{38}{50}$ $\frac{51}{60}$ $\frac{32}{40}$ $\frac{12}{15}$

8. Calculate 6% tax on each item.
   a)

   $89.50

   b)

   $39.99

What Do You Think Now?

Revisit What Do You Think? on page 153. How have your answers and explanations changed?

Chapter **Review**

Frequently Asked Questions

Q: **How do you solve a percent problem using a proportion?**

A: Write a proportion in which one of the fractions or ratios is out of 100. For example, suppose that a hockey team won 18 out of 40 games. To determine what percent of the games the team won, write $\frac{18}{40}$ in a proportion.

$$\overset{\times?}{\frac{18}{40} = \frac{\blacksquare}{100}} \underset{\times?}{}$$

Figure out what number you need to multiply 40 by to get 100.
Using division, $100 \div 40 = 2.5$.

$$\overset{\times 2.5}{\frac{18}{40} = \frac{45}{100}} \underset{\times 2.5}{}$$

Multiply the numerator and the denominator by 2.5.
$\frac{18}{40}$ is equivalent to 45%.
They have won 45% of the games.

Q: **How do you solve a percent problem using decimals?**

A: Rename the percent as an equivalent decimal. Then multiply the decimal by the given amount. For example, suppose that you want to determine 27% of 45.

Rename 27% as 0.27, and multiply 0.27 by 45.

$0.27 \times 45 = 12.15$

So, 27% of 45 is 12.15.

Q: **How can you solve a percent problem using logical reasoning?**

A: Use information you already know to make your calculations easier. For example, if you know that 30% is 20 items, then you can figure out 45%, which is 30% + 15%. Because 15% is half of 30%, and 10 is half of 20, 45% is 20 + 10 items. So, 45% is 30 items.

Practice

1. Rename each percent as a fraction.

   a) $55\% = \dfrac{\blacksquare}{20}$

   b) $26\% = \dfrac{\blacksquare}{50}$

   c) $66\% = \dfrac{\blacksquare}{50}$

   d) $5\% = \dfrac{\blacksquare}{40}$

2. Complete the table.

Percent	Decimal	Fraction (lowest terms)
70%		
	0.08	
		$\dfrac{3}{4}$
	0.15	
		$\dfrac{1}{4}$
55%		

3. Write each fraction as a percent.

   a) $\dfrac{1}{5}$

   b) $\dfrac{3}{4}$

   c) $\dfrac{9}{10}$

   d) $\dfrac{3}{12}$

4. In John's school, 20% of the students have no brothers or sisters, and 32% have one sibling.

   a) What percent of the students have more than one sibling?

   b) There are 355 students in the school. How many have no siblings?

Lesson 4.3

5. Estimate.

   a) 81% of 70

   b) 96% of 202

   c) 42% of 498

   d) 28% of 11

6. What benchmarks did you use for your estimates in the previous question?

7. In 2001, Statistics Canada found that 8% of Canada's population had asthma. When Marla surveyed her school, she found that 32 of the 301 students had asthma. Estimate if this number is greater or less than might be expected.

8. Why is it easier to calculate 5% of 230 than 8% of 230?

Lesson 4.4

9. Josip spent $18 of his $30 on a sweatshirt. Sanjeev spent $13 of his $20 on a CD. Who spent a greater percent of his money?

10. Complete each statement using <, >, or =.
a) $\dfrac{9}{50}$ ▪ $\dfrac{7}{40}$ **b)** $\dfrac{18}{60}$ ▪ $\dfrac{21}{75}$

Lesson 4.5

11. 14% of a restaurant's income is from dim sum on Saturdays. Last week, the restaurant made $4100. How much money came from dim sum on Saturday?

12. Determine each missing number.
a) 25% of 84 = ▪   **b)** 10% of ▪ = 5

13. A movie theatre has sold 75% of its seats for the 7:00 p.m. show. The theatre has 440 seats. How many tickets have been sold?

14. Renato sold a dining-room set for $144, which was 12% of its value when new. How much did the dining-room set originally cost?

Lesson 4.6

15. Calculate.
a) 79% of 20   **c)** $456 plus 15% tax
b) 40% of 182.8   **d)** $49.98 plus 6% tax

16. Dmitri scored 112 out of a possible 140 points in a video game. What is his percent score?

Lesson 4.7

17. a) Describe a quick way to determine 25% of 284.
b) Determine 75% of 284 using your answer to part (a). Explain.

18. The mass of a sperm whale is 42 t. This is about 70% of the mass of a fin whale. What is the mass of a fin whale?

19. Mark calculated 34% of 55. Explain each step of Mark's thinking.
10% is 5.5.   30% is 16.5.   2% is 1.1.   4% is 2.2.   34% is 18.7.

Chapter **Task**

Ball Bounce-ability

Have you ever dropped a ball to see how high it would bounce? Do different types of balls bounce better than others?

❓ How can you determine which type of ball bounces the best?

A. With a partner, choose three different types of balls. Each ball should be a different size and material. For example, a basketball, golf ball, tennis ball, and soccer ball are different sizes and made from different materials.

B. Create a table to record the type of ball, the drop height, the bounce height, the ratio of $\frac{\text{bounce height}}{\text{drop height}}$, and the percent.

C. Select a ball. Measure and record the height that the ball is dropped from and the distance that the ball bounces up from the floor on the first bounce.

D. Repeat part C for the other two balls.

E. Determine the ratio of $\frac{\text{bounce height}}{\text{drop height}}$ for each ball. Express this ratio as a percent.

F. Discuss the results of your experiment in a report, and rank the bounce-ability of each ball.