

Chapter


Linear Relations and Linear Equations

GOAL
You will be able to

- describe a pattern rule using a linear relation
- represent a linear relation using an algebraic expression
- evaluate an expression given the value of a variable
- describe the relationship between a pattern, its table of values, and its graph
- solve a problem represented by a linear equation and verify the solution

What is the relationship between the number of black stripes in the centre of the chief's blanket and the number of black border pieces? Is the relationship between the number of red stripes and the number of red border pieces the same?

Chapter 9

## Getting Started

## Finding Calendar Patterns



Q
What number patterns can you find in the calendar?
A. Copy the calendar page. Draw a box around any $2 \times 2$ square. Add the pairs of numbers along the diagonals. What do you notice?
B. Now choose two other $2 \times 2$ squares. Add the pairs of numbers along the diagonals. What pattern do you find?
C. Add the numbers in the columns in each of the three squares. What pattern do you find?
D. Add the numbers in the rows in each square. What pattern do you find?
E. Why do the patterns in the squares work the way they do?
F. Repeat parts A to E for $3 \times 3$ squares in the calendar.

## What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. To predict the number of cubes in figure 6 , you could use a table of values.

figure 1

figure 2

figure 3
2. When you write the formula for the area of a rectangle, you must always use $l$ and $w$ as the variables.
3. Starting with the equation $2 r=8$, you can create many different equations with the same solution.
4. A computer program gives the output shown for the inputs 1 to 3 . To know the output for 10 , you need to know the outputs for 1 to 9 .

| Input | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | 7 | 13 | 19 |  |  |  |  |  |  |  |

## Writing a Pattern Rule

## YOU WILL NEED

- coloured square tiles

GOAL
Write a pattern rule using numbers and variables.

## LEARN ABOUT the Math

Ryan made this pattern using coloured tiles.

## relation

a property that allows you to use one number to get information about another number.
For example, the perimeter of a square is 4 times the length of one side, so if you know the length of one side of the square, you can determine the perimeter. This relation can be represented by the formula $P=4 s$ or by a table of values.

| Side <br> length $(\mathrm{cm})$ | Perimeter <br> $(\mathrm{cm})$ |
| :---: | :---: |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |


figure 1
figure 2


figure 3
?
How can you write the pattern rule using numbers and variables?
A. Complete the table.

| Figure <br> number | Number of <br> green tiles | Number of <br> orange tiles | Total number <br> of tiles |
| :---: | :---: | :---: | :---: |
| 1 | 3 | 4 | 7 |
| 2 | 3 | 8 | 11 |
| 3 |  |  | 15 |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

B. Use words to describe the relation between the number of orange tiles in a figure and its figure number.

## constant term

a quantity that does not change; for example, in $2 \times n+5$, 5 is a constant term

## numerical coefficient

the number that is the
multiplier of a variable;
for example, in
$2 \times n+5$,
2 is the numerical coefficient of $n$
C. Represent the figure number using the variable $n$. Write an algebraic expression that tells how to calculate the number of orange tiles in figure $n$.
D. How many green tiles are in figure $n$ ?
E. Write an algebraic expression to represent the total number of tiles in figure $n$.
F. Identify the constant term and the numerical coefficient in your algebraic expression.
G. Why is your expression from part E a pattern rule?

## Communication ITP

- Sometimes an algebraic expression is just called an expression.
- When you multiply a variable by a number or another variable, omit the multiplication sign. For example, write $2 a$ instead of $2 \times a$, and write $a b$ instead of $a \times b$.
- A variable can be represented either by a capital letter or a lower-case letter; for example, $A=b+2$. You can choose any letter as a variable, but you may want to choose a letter that reminds you of the quantity it represents; for example, $n$ for figure number.


## Reflecting

H. How did looking at the coloured tiles help you to write your algebraic expression in part E?
I. What do the constant term and the numerical coefficient of your expression tell you about the pattern?
J. You have described the same relation with figures, a table of values, words, and a pattern rule. Which description do you prefer? Explain your choice.

## WORK WITH the Math

## Example 1 Writing a pattern rule

Write a pattern rule to represent the relation between the number of blocks in any figure in this pattern and its figure number, $n$.


## Example 2 Visualizing a pattern in different ways

Write a pattern rule using an algebraic expression for the number of tiles in any figure in this pattern.

figure 1 figure 2

figure 3 figure 4

## Oshana's Solution


figure 1

figure 2

figure 3

figure 4

I drew the pattern.
There are always 2 tiles in the base. I coloured them blue.

The top increases in each figure. I coloured the tiles in the top green.

I used $T$ to represent the number of tiles in a figure and n to represent the figure number.

I used the algebraic expression $2+n$ to create the pattern rule.

My pattern rule is $T=2+n$.

I wanted to relate the number of tiles to the figure number $n$.

The number of blue tiles is always 2 .
The figure number is the same as the number of green tiles.

The pattern rule says that the number of tiles in a figure is the number of blue tiles plus the number of green tiles.

figure 1

figure 2

figure 3

figure 4

I drew the pattern.
I coloured the vertical tiles green.
The number of vertical tiles increases in each figure.

There is always 1 tile remaining. I coloured the 1 remaining tile blue.

I used $T$ to represent the number of tiles in a figure and $f$ to represent the figure number.

I used the algebraic expression $f+1$ to create the pattern rule.

My pattern rule is $T=(f+1)+1$.

In each figure, the number of green tiles is 1 more than the figure number.

The last 1 represents the 1 blue tile in each figure.

## Example 3 Predicting a pattern rule

Write a pattern rule to represent the relation between the number of tiles in any figure in this pattern and its figure number, $n$.


Sarah's Solution


The number of tiles in each figure increases by 6 each time (2 blue, 2 purple, and 2 green). That's how the 6 -times table works too.

| $n$ | $6 n$ |
| :---: | :---: |
| 1 | 6 |
| 2 | 12 |
| 3 | 18 |


| Figure <br> number | Number <br> of tiles | $6 n$ | Difference |
| :---: | :---: | :---: | :---: |
| 1 | 9 | 6 | 3 |
| 2 | 15 | 12 | 3 |
| 3 | 21 | 18 | 3 |
| 4 | 27 | 24 | 3 |

I compared the number of tiles in each shape with $6 n$.

The number of tiles in figure $n$ is always 3 greater than $6 n$.

The rule for the number of tiles in figure $n$ is $T=6 n+3$.

I can say the rule as, "Multiply the figure number by 6 and then add 3."

figure 3

## A Checking

1. Write a pattern rule using an algebraic expression for the number of tiles in any figure in the pattern at the left.

figure 3
2. a) What stays the same and what changes in the tile pattern at the left?
b) Write a pattern rule in words.
c) Write a pattern rule using an algebraic expression for the number of tiles in any figure.
d) Identify the numerical coefficient and the constant term in your expression.

## B Practising

3. a) Complete the table for the pattern at the left.

| Figure number | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of triangles | 4 | 5 |  |  |  |

b) Describe in words the relation between the number of triangles in a figure and its figure number.
c) Write a pattern rule using an algebraic expression to represent this relation.
4. Anne, Sanjay, and Robert wrote different pattern rules for the same pattern. Explain each student's reasoning.

figure 1

figure 2

figure 3

Anne: "My pattern rule is $T=n+1+n$."

figure 1

figure 2

figure 3

Sanjay: "My pattern rule is $T=n+(n+1)$."

figure 1

figure 2

figure 3

Robert: "My pattern rule is $T=2 n+1$."

figure 1
figure 2 figure 3

## Reading Strategy

Complete an
Understanding What I
Read chart for each term you don't understand in question 8.
5. Kyle and Tynessa coloured the same pattern of tiles differently. Kyle's colouring:

figure 1

figure 2

figure 3

Tynessa's colouring:

figure 1

figure 2

figure 3
a) Write a pattern rule using an algebraic expression based on Kyle's colouring.
b) Write a pattern rule using an algebraic expression based on Tynessa's colouring.
c) Identify the constant term and the numerical coefficient in each expression.
d) Why must Kyle's and Tynessa's pattern rules be the same, even though their algebraic expressions look different?
6. a) Draw the next two figures in the tile pattern at the left.
b) Write a pattern rule using an algebraic expression for the number of tiles in any figure.
c) Identify the constant term and the numerical coefficient in your expression.
d) What do the constant term and the numerical coefficient tell you about how the pattern grows?
7. Is there a figure with exactly 257 tiles in each pattern? Explain.
a) $T=2 n+1$
b) $T=2 n+4$
8. Suppose that you have a pattern rule with an algebraic expression.
a) What does the constant term tell you about the pattern?
b) What does the numerical coefficient tell you about the pattern?

## CURIOUS MATH

## Creating Number Tricks Using Algebra

How can you model number tricks?
A. Try this trick.
Think of a one-digit number.
Multiply by 6 .
Get the picture.

$\square \square 00000$
$\square \square 00000 \square 00000$

Subtract 5.
Divide by 3 .
$3 n$
Here's the algebra!
$n$
$6 n$
$6 n+10$

$$
3 n+5
$$

n

What's the answer? Always your starting number
B. Try this trick.

Think of a one-digit number.
Multiply by 6 .
Add 10.
Subtract the original number.
Divide by 5 .
Get the picture.
$\square$

[10)000000
$6 a+10$

$5 a+10$

$a+2$
Subtract the original number.
Subtract the original number.
$a$
$6 a$

Here's the algebra!

What's the answer? Always 2 !

1. Finish the trick below so that the answer is always the original number.

Try this trick.
Think of a one-digit number.
Multiply by 10 .
Add 10.
Divide by $\quad$.

Get the picture.

[10)00000

Here's the algebra! y $10 y$
$10 y+10$

Subtract $\quad$.
2. Make up a trick of your own. Use pictures and algebra to show how it works.

Try it with a classmate.

## 9.2 Evaluating an Expression to Solve a Problem

## YOU WILL NEED

- a calculator

GOAL
Create and evaluate an expression to solve a problem.


## LEARN ABOUT the Math

Denis and Nayana are helping to organize a school trip for 260 students to Regina.

- Swift Buses charges $\$ 7$ per student.
- Zim Buses charges $\$ 6$ per student, plus $\$ 500$.

Denis and Nayana decide to calculate the charge for each company.

## ? Which company charges less?

## Example 1 Evaluating an expression in one step

Create an expression to represent what Swift charges for any number of students. Determine the charge for 260 students using your expression.

## Denis's Solution

I used $n$ to represent the number of students.
$7 n$
$=7(260)$
$=1820$

From the description, I knew that there would be a $\$ 7$ charge for each student.

To evaluate my expression, I substituted 260 for $n$ because there are 260 students.

Swift will charge $\$ 1820$ for 260 students.

## Communication ITP

- Use brackets when you substitute a number for a variable. This will prevent you from running numbers together. For example, to evaluate $2 a$ for $a=10$, write 2(10).
- Write each step in a calculation directly under the previous step. Line up the equal signs. This makes the calculation easier to read and check. For example,

$$
\begin{aligned}
2 a+5 & =2(10)+5 \\
& =20+5 \\
& =25
\end{aligned}
$$

## Example 2 Evaluating an expression in two steps

Create an expression to represent what Zim charges for any number of students. Determine the charge for 260 students using your expression.

Nayana's Solution

| Number of <br> students | Zim's <br> charge $(\$)$ | Pattern |
| :---: | :---: | :---: |
| 0 | 500 | $500+0(6)$ |
| 1 | 506 | $500+1(6)$ |
| 2 | 512 | $500+2(6)$ |
| 3 | 518 | $500+3(6)$ |
| 4 | 524 | $500+4(6)$ |

I used n to represent the number of students.
$500+6 n$
$=500+6(260)$
$=500+1560$
$=2060$
Zim will charge $\$ 2060$ for 260 students.
$\$ 1820$ is less than $\$ 2060$, so swift will charge less.

I created a table of values for Zim's charges. Then, I looked for a pattern in the values.

I saw that the table started at 500 and the value increased by 6 with each row.

Each value is 500 more than 6 times the number of students.

My expression has a constant term of 500 and a numerical coefficient of 6 .
To evaluate my expression, I substituted 260 for $n$ because there are 260 students.

## Reflecting

A. Each company's charge was described in words. How else did Denis and Nayana describe the relation between the charge and the number of students?
B. Why was Denis able to evaluate his expression with just one arithmetic operation, while Nayana needed two operations?

## WORK WITH the Math

## Example 3 Solving a patterning problem

## Jasmin made a pattern using

 blue pattern blocks. How many blocks are in figure 20?Solution

figure 1

figure 2

figure 3


Each figure starts with 2 blocks at the bottom.
Figure 1 has 3 blocks.
Figure 2 has 4 blocks.
Figure 3 has 5 blocks.
Each figure has 2 more blocks than its figure number.

Use $n$ to represent the figure number.

$$
\begin{array}{ll}
n+2 & \\
=(20)+2 & \text { Substitute } 20 \text { for } n . \\
=22 & \text { There are } 22 \text { blocks in } \\
\text { figure } 20 .
\end{array}
$$

## A Checking

1. Evaluate each expression for $d=5$.
a) $6 d$
b) $5 d-1$
c) $d+1$
d) $3(d+2)$
2. Write an expression to represent the cost to rent a sled for a base fee of $\$ 35$, plus $\$ 12$ per hour.

## B Practising

3. Evaluate each expression for $a=3$ and $b=5$.
a) $3 a$
b) $8 b$
c) $9 a$
d) $2(b-1)$
4. Aisha did $t$ sets of weightlifting, except the last time when she stopped 2 reps from the end of the last set. There are 8 reps in each set.
a) Write an expression to represent how many reps Aisha did.
b) Suppose that $t=4$. Calculate how many reps she did.
5. Evaluate the expression $6(b-1)+3$ for $b=4$. Show and explain all the steps.
6. Write an expression for each cost.
a) $\$ 4$ to sharpen each pair of skates for a class skating party
b) hamburgers at $\$ 3$ each
c) $\$ 2$ per hour plus $\$ 5$ to rent skates
d) hats on sale for $\$ 10$ each
7. Each figure in the pattern is made from green, blue, purple, and yellow tiles. Each expression tells the number of tiles of one colour in a figure. Match each expression to its colour.
A. $2 n$
B. $2 n+4$
C. $2 n-1$
D. $n+2$

figure 1

figure 2

figure 3

figure 4
8. Samantha works in the snack bar at a community centre. She earns $\$ 8$ an hour. One weekend, she was paid a bonus of $\$ 50$.
a) Write an expression to represent her total earnings for $h$ hours.
b) On that weekend, she worked for 15 h . Calculate how much she earned. Show your work.
9. Jerry sells wool caps at a booth. He earns $\$ 25$ a day, plus $\$ 2$ for each cap that he sells. On Monday, he sold 17 caps.
a) Create a table of values to show Jerry's daily earnings when he sells $0,1,2,3,4$, and 5 caps.
b) Write an expression to describe Jerry's daily earnings.
c) Calculate how much money Jerry earned on Monday.
10. A box of DVDs costs $\$ 56$. Winnie can use $\$ 3$ coupons to reduce the price.
a) Write an expression to describe the amount that Winnie would pay if she used $c$ coupons.
b) Suppose that Winnie has 5 coupons. How much will she pay?
11. a) Complete the table.

b) Write a rule for this pattern. Use $n$ for the term number in your expression.
c) Predict the value of term 8 .
12. A banquet hall charges a flat rate of $\$ 1000$, plus $\$ 30$ per guest. Suppose that there is a party at the banquet hall every night. Why would the owner of the banquet hall find it useful to create an algebraic expression to calculate the bill for each party?

## Exploring Possible Values

YOU WILL NEED

- pattern blocks


## GOAL

Explore solving a relation with more than one variable.

## EXPLORE the Math

This chart shows the cost of different groups of pattern block shapes. Your class wants to buy 100 of each shape.


?
How much does each shape cost? How much will your class need to spend?

## Linear Relations and Their Graphs

## YOU WILL NEED

- grid paper
- a ruler
- triangular pattern blocks


## GOAL

Graph a linear relation and describe properties of the graph.

## LEARN ABOUT the Math

Megan is designing straight paths for the school garden. She thinks there is a relation she can use to predict how many paving stones and how many border pieces she will need for each path. She will need 12 paving stones for the first path.
1 paving stone
2 paving stones
3 paving stones
4 paving stones

6 border pieces

8 border pieces


10 border pieces


12 border pieces


How many border pieces will Megan need for a path with 12 paving stones?
A. Megan started a table of values to show how the number of border pieces changes as she adds paving stones. Complete the table.

| Number of paving stones | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of border pieces | 6 | 8 | 10 |  |  |  |  |  |

B. Write a relation that tells how to calculate the number of border pieces when you know the number of paving stones.

## linear relation

a relation whose plotted points lie on a straight line

## WORK WITH the Math

## Example 1 Graphing a linear relation using a table

The rule $T=4 n+3$ describes the relation between the figure number $n$ and the number of tiles in the figure in this pattern.
a) Graph the relation. Why can you say it is a linear relation?
b) Describe some of the

figure 1 figure 2
figure 3 patterns in the graph. Relate these pattterns to the pattern rule.

## Ryan's Solution

a)



I made a table of values.
I used $T=4 n+3$ to extend the table.
I used the values as coordinates for the points on the graph.

The data points fall along a straight line, so the relation is linear.


Each time the figure number increased by 1 unit, the graph increased by 4 units. This was because each figure increased by 4 square tiles from the figure before it. There are no points between the points I plotted, because I can't use part of a paving stone.

I extended the pattern to make a point on the vertical axis at 3 .
There is no figure number 0 , but I imagine it would look like this: These three tiles are represented by the constant term.

## Example 2 Solving a problem using a graph

A new factory makes golf carts. The graph shows the number of carts made each day. If this pattern continues, how many carts will the factory make on day 15 ?

Sarah's Solution

Golf Cart Schedule



Day number, $d$

I used the graph to create a pattern rule. I used c to represent the number of carts and $d$ to represent the day number. $c=6 d+$ ?

| Day. | Value | Value | Value of |
| :---: | :---: | :---: | :---: |
| $d$ | of $6 d$ | from graph | $6 d+3$ |
| 1 | 6 | 9 | 9 |
| 2 | 12 | 15 | 15 |
| 3 | 18 | 21 | 21 |
| 4 | 24 | 27 | 27 |

$c=6 d+3$
$=6(15)+3$
$=93$

The points are in a straight line, so I think the other points will be on the same line.

There are 6 more golf carts made each day.
So I used a table to compare the value of $6 d$ to the value on the graph.
The values on the graph are 3 greater than the values of $6 d$. I tried $c=6 d+3$ by adding another column to the table. The values match.

I substituted 15 into the pattern rule.
On day 15 , the factory will make 93 golf carts.

## Example 3 Matching a linear relation to its graph

## Match each linear relation to its graph.

Relation A: $y=2 x+3 \quad$ Relation $\mathrm{B}: ~ y=3 x+2$


Graph B


Graph C


## Jacob's Solution

I made a table of values for each relation.

| $x$ | $2 x+3$ | $x$ | $3 x+2$ |
| :---: | :---: | :---: | :---: |
| 1 | 5 | 1 | 5 |
| 2 | 7 | 2 | 8 |
| 3 | 9 | 3 | 11 |
| 4 | 11 | 4 | 14 |
| 5 | 13 | 5 | 17 |



For $y=2 x+3$, the coordinates are $(1,5),(2,7)$, $(3,9),(4,11)$, and $(5,13)$. These are the points on graph B, so relation A matches graph B.


For $y=3 x+2$, the coordinates are $(1,5),(2,8)$, $(3,11),(4,14)$, and $(5,17)$. These are the points on graph A, so relation B matches graph A.

## A Checking

1. Graph the points in this table of values.

| $b$ | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| $2 b-2$ | 0 | 2 | 4 | 6 |

2. a) Create a table of values for the relation $y=n+3$.
b) Graph $y=n+3$ using your table of values.

## B Practising

3. Create a table of values for each linear relation.
a) $y=3 n+1$
b) $y=5 n+3$
c) $y=3 n+5$
4. Graph each linear relation in question 3 using your tables of values.
5. Graph these linear relations on the same set of axes. Use a different colour for each relation.
a) $y=2 n+5$
b) $y=2 n+7$
c) $y=2 n+3$
d) $y=2 n+1$
6. How are the graphs in question 5 alike? How are they different?
7. Graph these linear relations on the same set of axes. Use a different colour for each relation.
a) $y=n+3$
b) $y=2 n+3$
c) $y=3 n+3$
d) $y=4 n+3$
8. How are the graphs in question 7 alike? How are they different?
9. Match each linear relation to its graph.
A. $y=b+3$
B. $y=2 b+4$
C. $y=2 b$

## Graph A



## Graph B



Graph C

10. Mohammed is building a fence. Each section has 3 horizontal rails connecting 2 vertical posts.

$$
\left|=\sum\right|
$$

a) Write a rule for the number of rails Mohammed will need, based on the number of posts.
b) Graph the relation described by your pattern rule.
c) Mohammed's fence will have 10 posts. Determine how many rails he will need using your graph.
11. Mohammed is building another fence.

a) How is the pattern rule for this fence different from the pattern rule for the fence in question 10 ?
b) How are the graphs different?
12. a) Write a pattern rule to represent this toothpick pattern.
figure 1
figure 2


figure 3

figure 4

figure 5
b) Graph the relation between figure $n$ and the number of toothpicks needed to make the figure.
13. a) Write a pattern rule for the pattern $6,11,16,21,26, \ldots$ that relates each term to its term number.
b) Graph the relation between the terms and their term numbers.
14. Kaitlyn is making a flower-bed border with triangular paving stones and border pieces, like those Megan used for her path. Kaitlyn drew this graph.

Flower-Bed Border

a) What relation does the graph show?
b) Write a pattern rule for this relation.
c) How can you determine the length of each border piece?
15. a) Draw a pattern of tiles that matches each rule.
A. $y=n+5$
B. $y=5 n$
C. $y=3 n+1$
D. $y=2 n+3$
b) Graph each pattern rule.
c) Describe how the constant term and the numerical coefficient of the variable in each expression affect the graph and the figures in the geometric pattern.
16. A pattern rule can be written using an expression like $x+$. How will the graph of the rule be affected by changing the values of $\square$ and ?

## Mid-Chapter Review

## Frequently Asked Questions

Q: How do you evaluate an expression?
A: Substitute a number for the variable, and then use order of operations to calculate. For example, you can evaluate $2 n+1$ for $n=4$ as follows:

$$
\begin{aligned}
2 n+1 & =2(4)+1 \\
& =8+1 \\
& =9
\end{aligned}
$$

Q: How do you graph a linear relation?
A: Make a table of values, and then use it to plot points on a scatter plot. One variable tells the horizontal position of the point, and the other variable tells its vertical position. For example, graph $f=3 n+2$.


Q: How can you use the plotted points of a linear relation to determine other points of the relation?

A: Place a ruler along the plotted points. Other points in the relation will also lie on the line made by the ruler. For example, for the graph of $f=3 n+2$, you can use a ruler to see that $(4,14)$ and $(6,20)$ are also points in the relation.

## Practice

## Lesson 9.1

1. Write a pattern rule using an algebraic expression for the number of tiles in any figure in this pattern.

| Figure <br> number | Picture | Number <br> of tiles |  |
| :---: | :---: | :---: | :---: |
| 1 | $\square$ | $\square$ | 3 |
| 2 | $\square$ | $\square$ | $\square$ |
|  | $\square$ | $\square$ | 6 |
| 3 | $\square$ | $\square$ | $\square$ |

## Lesson 9.2

2. a) Draw the next two figures in this pattern.

figure 1

figure 2

figure 3
b) Write a pattern rule using an algebraic expression for the number of tiles in any figure in this pattern.
c) Predict the number of tiles used in figure 10.
3. Emma and Alex coloured the same pattern of tiles differently. Emma's colouring:

figure 1

figure 2

figure 3

Alex's colouring:

figure 1

figure 2 figure 3
a) Write a pattern rule using an algebraic expression for each colouring.
b) Identify the constant term and the numerical coefficient in each expression.
c) Describe what the constant term and the numerical coefficient in each expression tell you about the pattern.
4. The cost to rent skates is $\$ 3$, plus $\$ 2$ per hour.
a) Write an expression that tells the cost to rent skates for any number of hours.
b) Determine the cost to rent skates for 8 h .

## Lesson 9.4

5. The table of values and the graph show the relation between the hours that Erin works and her earnings. Enter the missing numbers in the table of values.

6. Graph these linear relations on the same set of axes. Use a different colour for each relation.
a) $y=3 n+2$
b) $y=3 n-1$
c) $y=3 n$
7. How are the graphs in question 6 alike? How are they different?
8. a) Write a pattern rule for the number of toothpicks in each figure.

figure 1

figure 2

figure 3
b) Graph the relation described by the rule.
c) Predict the number of toothpicks in figure 7 using the graph.

## 9.5

## YOU WILL NEED

- toothpicks


## Solving Equations Using Mental Math

## GOAL

Solve a problem by solving a related equation.

## LEARN ABOUT the Math

Megan and Oshana are building this toothpick pattern. They have 28 toothpicks.

figure 1

figure 2

figure 3

## equation

a statement that two quantities or expressions are equivalent; for example, $4+2=6$ and $6 x+2=14$
solution to an equation
a value of a variable that makes an equation true; for example, the solution to
$6 x+2=14$ is $x=2$

## ? What is the largest figure they can build?

A. Write a pattern rule for the number of toothpicks in figure $n$.
B. Write an equation to represent the problem.
C. Your equation may look like $\square \times$ variable $+\square=28$. What is the value of the $\square \times$ variable part?
D. What is the value of your variable?
E. Which figure has 28 toothpicks?
F. Check your solution to the equation.

## Reflecting

G. How are your pattern rule in part A and your equation in part B alike? How are they different?
H. Think about your equation. What meaning does each part of the equation have when you connect it to the problem?
I. How did you use mental math to calculate the solution?

## WORK WITH the Math

## Example 1 Solving a problem using an equation

Irene breeds and sells rabbits. One day, she sold half of her rabbits. Then, she sold one more. She still had 13 rabbits. How many rabbits did she have to start?

## Denis's Solution

I used $r$ to represent the number of rabbits she started with.
$\frac{r}{2}-1=13$
$\frac{r}{2}=14$
$r=28$
Verify the solution:
Suppose that Irene started with 28 rabbits. If she sold half, she would have 14 left. If she sold 1 more, she would have 13. This matches what the problem said.
$\frac{r}{2}$ represents the number of rabbits left after half are sold. I still need to subtract 1 for the rabbit sold later.
After all the rabbits were sold, there were 13 left.

If I subtract 1 from something and get 13 , I must have started with 14 . This means that $\frac{r}{2}$ must equal 14 .

The number you divide by 2 to get 14 is 28 .
I substituted my solution back into the original problem.

The solution $r=28$ is correct. Irene started with 28 rabbits.

## Example 2 Solving a sharing problem using an equation

Scott and his 4 friends mowed lawns together last summer. They split their earnings equally, and each person received $\$ 75$.
Represent the situation with an equation. Determine the total amount of money they earned using your equation.

## Nayana's Solution

$\frac{m}{5}=75$
$m=5 \times 75$
$\mathrm{m}=375$
The friends earned $\$ 375$.

I used $m$ to represent the money that was shared. The friends divided the money 5 ways.

The 5 friends each got $\$ 75$, so the total amount is 5 times $\$ 75$.

## A Checking

1. Solve. Verify your solutions.
a) $n+6=13$
b) $w-11=22$
c) $9 p=63$
d) $2 n+3=15$
e) $\frac{x}{3}=9$
f) $\frac{y}{4}=7$
2. Jamie wants to know which figure in this pattern has exactly 43 tiles.

figure 1

figure 2

figure 3
a) Write an equation to represent Jamie's problem.
b) Solve the equation.
c) Check your solution. Show what you did.

## B Practising

3. Solve each equation.
a) $7 b=84$
b) $11=q-4$
c) $8+z=30$
d) $\frac{w}{10}=20$
e) $22=m+2$
f) $35=n-5$
4. Solve each equation.
a) $23=2 m+3$
b) $42=6 a-6$
c) $9 n-4=32$
d) $5 n+5=40$
5. a) Explain each step in this solution.

$$
\begin{array}{rlrl} 
& & 6+5 m & =16 \\
\text { Step 1: } & 5 m & =10 \\
\text { Step 2: } & m & =2
\end{array}
$$

b) How can you verify that the solution $m=2$ is correct?
6. Look again at Jamie's pattern in question 2. Can you build a figure in this pattern with exactly 39 tiles? Explain.
7. a) Write a pattern rule to represent the number of tiles in each figure in this pattern.

figure 1

figure 2

figure 3

figure 4
b) Albert wants to build the figure with 24 tiles in this pattern. Write an equation you can solve to determine the number of the figure with 24 tiles.
c) Solve your equation.
d) Verify your solution by drawing the figure and counting the tiles.
e) Determine which figure in this pattern has 21 tiles, using your equation. Describe the figure with 21 tiles.
8. a) Loretta wants to build the figure with 28 pattern blocks in the following pattern. Write an equation you can solve for the number of this figure.

figure 1

figure 2

figure 3
b) Solve your equation and verify your solution.

!

figure 1 figure 2

figure 3
9. a) Wilmer wants to build the figure with exactly 30 tiles in the pattern at the left. Write an equation you can solve for the number of the figure.
b) Solve your equation and verify your solution.
10. To rent a movie, a company charges $\$ 2$, plus $\$ 1$ per day.
a) Write a pattern rule to represent the cost, $c$, of renting a movie for $d$ days.
b) Write an equation to answer the question, "If you have $\$ 9$, for how many days can you rent a movie?"
c) Solve your equation and verify your solution.
11. Susan has $\$ 25$. She is going to spend $\$ 4$ on a book, and then $\$ 3$ per day on lunch.
a) Write an equation to answer the question, "For how many days can Susan buy lunch?"
b) Solve your equation and verify your solution.
12. Kevin says to Zach, "I am thinking of a number. If you double it and then subtract 1 , the result is $7 . "$
a) Write an equation to determine Kevin's number.
b) What steps can you use to solve the equation? Explain.
13. What is the greatest number of squares you can build with 100 toothpicks using this pattern?

figure 1

figure 2

figure 3
14. What is the greatest number of triangles you can build with 100 toothpicks using this pattern?

figure 1

figure 2

figure 3

figure 4
15. Look at the linear relation $\square x+\bigcirc=$. Suppose that you know the value each geometric symbol represents. What steps would you use to solve for $x$ using mental math?

## 9.6 Solving Equations Using Models and Drawings

## YOU WILL NEED

- integer tiles, coloured counters, or coloured cubes


## GOAL

## Model and solve problems using pictorial and concrete methods.

## LEARN ABOUT the Math

Sarah showed her classmates the following number trick:

1. Pick a whole number.
2. Multiply the number by 2 .
3. Add 5 to the result.

Jacob said, "My answer is 17. "
Sarah used the equation $2 n+5=17$. She said, "You started with 6." She was correct.


## Example 2 Using a drawing

Represent the steps in Sarah's solution using a drawing.

## Megan's Solution

| n | , | 5 |
| :---: | :---: | :---: |
| 17 |  |  |
| $\downarrow$ |  |  |
| n | n | 5 |
| 12 |  | 5 |
|  | $\downarrow$ |  |
| n | n | 5 |
| 6 | 6 | 5 |

I drew identical rectangles to represent each side of the equation. Then I split one to show the two $n$ terms and the constant term, 5 .

The two $n$ pieces had to have a value of $17-5=12$.

Each $n$ had to have a value of 6 .
So, $n=6$.

## Reflecting

A. How do Jacob's model and Megan's drawing show each step needed to solve the equation?

## WORK WITH the Math

## Example 3 Modelling and solving integer equations

Model and solve each equation.
a) $x+(-2)=(-3)$
b) $x+(-2)=(+3)$
c) $x+(+2)=(-3)$

## Solution

a) $x+(-2)=(-3)$

b) $x+(-2)=(+3)$
c) $x+(+2)=(-3)$
$x+\bigcirc \bigcirc=\bigcirc \bigcirc$
$x+\bigcirc \bigcirc=\bigcirc \bigcirc$
$x+\bigcirc \bigcirc \bigcirc \bigcirc=\bigcirc \bigcirc \bigcirc \bigcirc$
$\begin{aligned} x+0000 & =00000 \\ x & =00000\end{aligned}$ $x=(+5)$
$x+\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$
 $x=(-5)$

## Example $4 \mid$ Solving an equation by dividing

Tom shared his candy with 4 friends. Everyone got 3 pieces. Represent the situation with an equation.
Solve the equation to determine the number of pieces that Tom began with.

## Solution



Use $x$ to represent the total number of pieces of candy.
Draw 5 columns, because each person got $\frac{1}{5}$ of the candy.

| $\frac{x}{5}$ | $\frac{x}{5}$ | $\frac{x}{5}$ | $\frac{x}{5}$ | $\frac{x}{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 3 | 3 | 3 | 3 |
| $\checkmark$ |  |  |  |  |
| $x$ |  |  |  |  |
| 15 |  |  |  |  |

Each person got 3 pieces.
The equation $\frac{x}{5}=3$ represents the situation.

Tom began with 15 pieces of candy.

## A Checking

1. Represent each equation using a drawing or a model with counters. Then solve each equation.
a) $p-9=15$
b) $5 w-2=13$
c) $z+(-5)=(-2)$

## B Practising

2. Represent and solve each equation.
a) $p+8=10$
b) $2 h+6=12$
c) $x-7=4$
d) $\frac{c}{6}=2$
e) $x+(-5)=(-1)$
f) $y+(-4)=(-3)$
3. a) Write a pattern rule for the number of counters in figure $n$ of the pattern shown at the left.
b) Write an equation to solve the problem, "Which figure has exactly 73 counters?"
c) Solve your equation. Show what you did.
4. Write each sentence as an equation. Then solve each equation.
a) The sum of a number and 19 is 35 .
b) Eight times a number is 192.
c) When you multiply a number by 9 and subtract 16 , the result is 47 .
5. Andrea has $\$ 73$ in her bank account. She takes out $\$ 5$ every week.
a) Write a linear relation to show the amount of money in Andrea's account after $w$ weeks.
b) Write an equation to represent the number of weeks before Andrea has $\$ 58$ left in her account.
c) Solve your equation. Show what you did.
6. a) Write an equation for the number of cubes in figure $n$ of the pattern shown at the left.
b) Which figure has exactly 25 cubes?
c) Does any figure have exactly 16 cubes? Explain.
7. Angelica has 61 toothpicks. What is the largest figure that she can build in this pattern?
figure 3


figure 2

figure 3
8. A computer program gives the output shown for the inputs 1 to 3 .

| Input | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | 7 | 13 | 19 |  |  |  |  |  |  |  |

a) Write the calculation rule that the program uses.
b) Write an equation to answer the question, "What input will give an output of 61 ?"
c) Solve the equation. Show what you did.
9. Suppose that you have to solve an equation. When is it better to use a drawing to represent the solution? When is it better to use concrete materials, such as counters?

## MATH GAME

## The Number Game

The goal is to guess what expression another player has made.
Number of players: 2 to 4

## How to Play

1. Take turns being the Rule Maker. The Rule Maker writes an expression with one variable, but does not show it to the other players.
2. The Rule Maker asks a player for a number. The Rule Maker substitutes this number into the expression and tells the result. The player who gave the number then tries to figure out the rule.
3. If the rule is incorrect, the Rule Maker asks the next player for a number.
4. The Rule Maker continues to ask for numbers until a player figures out the rule.
5. Play until everyone has been the Rule Maker. The player with the greatest score wins.

## Scoring

1. The Rule Maker receives 1 point for each incorrect rule.
2. The player who figures out the rule receives the same number of points as the Rule Maker for that round, plus 10 extra points.


Nayana was the Rule Maker.
Denis said 2, and the result was 7 . Denis's rule was incorrect.
Jacob said 3, and the result was 9 . Jacob's rule was incorrect.
Then it was my turn.
I said 4, and the result was 11 .
I figured out that the expression was $2 n+3$. I was right!
Nayana got 2 points, and I got $2+10=12$ points.

# 9.7 Solving Equations by Graphing 

## YOU WILL NEED

- grid paper
- a ruler

GOAL
Model and solve problems using tables of values and graphs.

## LEARN ABOUT the Math

Every week, Denis deposited $\$ 2$ in his bank account. On his birthday, he deposited an extra $\$ 15$. His new bank balance was $\$ 105$.

## How many deposits did Denis make?



## Example 2 Drawing a graph

Determine how many deposits Denis made using a graph.
Sarah's Solution


I made a table of values.


Then, I graphed the points.
The balance was $\$ 105$, so I drew a line from 105 across the graph. I put the ruler along the plotted points and looked for the intersection. I drew a new point there.

I drew a line down from the new point on the graph.

The line met the $d$-axis at 45 .
Denis made 45 deposits.

## Reflecting

A. How does Sarah's method of drawing a horizontal line and then a vertical line solve the equation?

## WORK WITH the Math

## Example 3 Solving a patterning problem

Determine which figure in this pattern has exactly 39 tiles, using a graph.

## Solution


figure 1
figure 2
figure 3

Use $n$ to represent the figure number.

| Figure number <br> (term number) | Number of tiles <br> (term value) |
| :---: | :---: |
| 1 | 7 |
| 2 | 11 |
| 3 | 15 |
| 4 | $4 n+3$ |
| $n$ |  |

Figure 9 has 39 tiles.

Graph the pattern rule $4 n+3$.


## A Checking

1. Solve $2 n+3=17$ using the graph.

## B Practising

2. a) Make a table of values for the pattern at the left.
b) Write a pattern rule for the number of tiles in each figure.
c) Write an equation to determine which figure has 22 tiles.
d) Solve your equation.
3. a) Make a table of values for this pattern.

figure 1

figure 2

figure 3
b) Write a pattern rule for the number of counters in each figure.
c) Write an equation to determine which figure has 23 counters.
d) Solve your equation.
4. This graph shows David's bank balance.

Weekly Bank Balance

a) Make a table of values from the graph.
b) Create a pattern rule to represent David's bank balance at $w$ weeks.
c) Write an equation to determine when his bank balance is $\$ 60$.
d) Solve your equation.
e) When will David's bank balance be $\$ 100$ ?
f) What will it be after 20 weeks?
5. Which figure in each pattern has exactly 97 toothpicks?
a)

figure 1

figure 2

figure 3
b)

figure 1

figure 2

figure 3

figure 4
6. Graph these pattern rules on the same set of axes. Use a different colour for each rule. Solve each rule for $r=17$ using your graph.
a) $r=2 n+5$
b) $r=2 n+7$
c) $r=2 n+9$

7. Rowyn can rent an electric lawn mower for $\$ 100$. She charges $\$ 15$ to mow a lawn. She wants to know how many lawns she needs to mow to earn enough money to buy a used guitar and amplifier priced at $\$ 425$.
a) Graph this situation. Put the number of lawns Rowyn might mow on the horizontal axis and the amount she will earn on the vertical axis.
b) Write an expression that relates Rowyn's earnings to the number of lawns she mows.
c) Solve an equation to determine how many lawns Rowyn needs to mow.
8. A grocery store collects donated canned food for a food bank. The cans are packed in boxes that hold 24 cans. The store has collected 744 cans. How many boxes does the store need?
a) Graph this situation. Put the number of boxes on the horizontal axis and the number of cans on the vertical axis.
b) Determine the number of boxes needed using your graph.
c) Write an expression to relate the number of boxes needed to the number of cans.
d) Solve an equation to determine the number of boxes needed.
e) You have solved this problem in two ways. How else could you have solved it?
9. Suppose that an equation looks like $x+\bigcirc=12$. Describe how you could use the graph of $y=\square x+$ to solve the equation.

## 9.8 Communicate the Solution of an Equation

## YOU WILL NEED

- counters
- lightweight containers


## GOAL

Use models, words, and symbols to record and explain the steps in solving an equation.

## LEARN ABOUT the Math



Jacob is using a drawing to represent the steps needed to solve $4 n+3=15$. Megan is asking questions to help Jacob improve his explanation.

## How can Jacob improve his explanation?

## Jacob's Explanation

I represented the equation with a picture of a balance scale.


I represented each step in the solution with more pictures.

## Megan's Questions

How did you know what to put on each side of the balance scale?


This picture represents $4 n+3-3=15-3$.


Communication

## Checkifst

$\checkmark$ Did you show each step in your reasoning?
$\checkmark$ Did you use a model or picture to illustrate or justify your reasoning?
$\checkmark$ Did you record your solution using mathematical symbols?
$\checkmark$ Did you use words and pictures that were clear and correct?
A. Use Megan's questions and comments to improve Jacob's explanation.
B. What parts of the Communication Checklist did Megan deal with in her questions?

## Reflecting

C. How does using a balance model make Jacob's explanation easier to understand?
D. Suppose that Jacob had used a real balance scale, real containers, and real counters to demonstrate his solution. Why might his demonstration not work out exactly?
E. Which part of Jacob's solution was most difficult to explain? Why?

## WORK WITH the Math



## A Checking

1. Use a balance model and the Communication Checklist to improve this solution.

$$
\begin{aligned}
& 3 c+1 & =10 \\
\text { Step 1: } & 3 c & =9 \\
\text { Step 2: } & c & =3
\end{aligned}
$$

## B Practising

2. Draw a balance model to illustrate each step used to solve this problem.

$$
\begin{aligned}
4 c & =12 \\
c+c+c+c & =3+3+3+3 \\
c & =3
\end{aligned}
$$

3. Write and solve an equation for each balance problem.

Explain each step.

4. Solve each equation. Explain each step.
a) $3 x+5=11$
b) $x+(-5)=(-1)$
c) $2 z=8$
d) $\frac{a}{5}=10$
e) $(-8)=b+(+3)$
f) $19=4 b+3$
5. Why is it important to explain your steps clearly when describing how you solved an equation?

## Chapter Self-Test

1. Write a rule for this pattern.

figure 1

figure 2

figure 3
2. A membership in a store's book club costs $\$ 10$. Members can buy paperbacks for $\$ 6$ each.
a) Write an expression for the cost of paperbacks for a book club member, including the cost of a membership.
b) Write the equation that you should solve to determine the number of books a member can buy for $\$ 106$.
c) Solve your equation. Show each step.
3. Evaluate $9 a+7$ for $a=5$. Show your work.
4. Solve $5 a+2=17$. Show each step.
5. Solve $3 n+7=52$ using the graph at the left.
6. Which figure in this pattern has exactly 161 counters?

figure 1

figure 2

figure 3
7. Solve each equation. Explain each step.
a) $6 x=78$
b) $n+(-2)=(+5)$
c) $6 p+1=19$
d) $\frac{r}{5}=10$

## What Do You Think Now?

Revisit What Do You Think? on page 375. How have your answers and explanations changed?

## Chapter Review

## Frequently Asked Questions

Q: What is the difference between an expression and an equation?

A: An expression ...
... is like a word phrase.
... may contain one or more operations but does not have an equal sign.
... can be evaluated by substituting a number for each variable and then calculating.

Examples: $3 n, b+4$, $2 p-7$

An equation ...
... is like a word sentence.
... may contain one or more operations and does have an equal sign.
... can be solved by determining the values of the variables that make the equation true.

Examples: $3 n=6$, $b+4=13,2 p-7=37$

## Q: How can you solve an equation?

A1: You can use mental math and reason out the solution by working backward. For example, to solve $26=3 b+5$, you can reason as follows:

- Since you add 5 to $3 b$ to get $26,3 b$ must be 21 .
- Three times a number is 21 , so the number must be 7 .
- Verify your answer by substituting the value you got for the variable into the original equation. Both sides of the equation should have the same value. If not, try again.

$$
\begin{aligned}
& 26=3 b+5 \\
& \text { Verify. } \\
& 21=3 b \\
& 7=b \\
& \text { Left side: Right side: } \\
& 26 \\
& 3 b+5 \\
& =3(7)+5 \\
& =21+5 \\
& =26
\end{aligned}
$$

A2: You can illustrate each step with a model or drawing. For example, to solve $3 n+2=17$, you can use a balance model.
 $n$ represents the number of counters in each bag. $3 n+2=17$


$$
3 n+2-2=17-2
$$


$3 n=15$


Graph of $2 x+3$
$n=5$
Each bag contains 5 counters.

A3: You can use a table of values and a graph. For example, to solve $2 x+3=25$, make a table of values. The coordinates of the points are $(1,5),(4,11)$, and $(7,17)$. Plot these points.

| $x$ | 1 | 4 | 7 |
| :--- | :---: | :---: | :---: |
| $2 x+3$ | 5 | 11 | 17 |

Draw a line from 25 on the vertical axis. Place a ruler beside the dots. Put a dot where the ruler and the 25 line meet. Draw a line down from this point to the horizontal axis. The value on the horizontal axis is the solution. From the graph, $x=11$.

## Practice


figure 1 figure 2 figure 3
figure 1

figure 2

figure 3

Graph of $2 n+4$



## Lesson 9.1

1. Write a pattern rule using an algebraic expression for the number of toothpicks in any figure in the pattern at the left.

## Lesson 9.2

2. A tool rental company rents a spray painter for a flat rate of $\$ 25$, plus $\$ 5$ per hour.
a) Write an expression to represent the cost to rent the spray painter for any number of hours.
b) Determine the cost to rent the spray painter for 9 h .

## Lesson 9.4

3. a) Write a pattern rule to represent the counter pattern at the left.
b) Graph the relation that the rule represents.
c) Determine the number of counters in figure 5 using your graph.

## Lesson 9.5

4. A basketball team has to raise $\$ 900$ for new uniforms. The players have raised $\$ 300$ from food sales. To raise the rest, they are holding a shoot-out challenge. In all, 25 teams have signed up. How much should each team pay?
a) Create an equation to represent this problem.
b) Solve your equation using mental math.
c) Check your solution. Show what you did.

## Lesson 9.6

5. Solve each equation.
a) $2 x+1=13$
b) $3 a+4=19$
c) $4 w+3=15$
d) $c+(+3)=(-5)$
e) $2 w+8=24$
f) $3 x+10=25$

## Lesson 9.7

6. Solve $2 n+4=20$ using the graph. Explain what you did.

## Task | Checklist

$\checkmark$ Did your drawings show clearly how your pattern worked?
$\checkmark$ Did you record your solution using mathematical symbols?
$\checkmark$ Did you check your equation and your solution?
$\checkmark$ Did you show each step in your reasoning?
$\checkmark$ Did you make sure that your solution was clear and easy to understand?

## Names, Patterns, and Equations

In art class, students are making mosaic tile patterns based on the initials in their names.

## ? What patterns can you create using one of your initials as a starting figure?

A. Build one of the letters in your name using linking cubes or coloured square tiles. (This will be the starting figure in your pattern.)
B. Write a rule that tells the number of tiles needed for each figure in your pattern. Use the rule to build the next two figures in your pattern.
C. Use colour to show which parts of your pattern rule change with every figure and which part stays the same.
D. Pose a question about your pattern that can be answered by solving an equation. Write the equation.
E. Solve the equation to answer your question. Illustrate each step in your solution with a drawing or model.
F. Show that your solution to the equation is correct.

