Chapter

6

Addition and Subtraction of Integers

GOAL

You will be able to

- represent addition and subtraction of integers using concrete materials, drawings, and number lines
- record the addition or subtraction of integers symbolically
- solve problems that involve the addition and subtraction of integers

Why should a pilot know how to read and use integers?

Getting Started

Interpreting Data

Five students in Tynessa's class in Lethbridge had birthdays in January. They wondered whose birthday was on the coldest day.

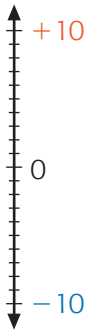
Student	Birthday	Low temperature on birthday (°C)
Aaron	January 1	−7
Kelly	January 6	4
Charles	January 11	−6
Janet	January 17	−3
Fred	January 25	−8

integer

the counting numbers (+1, +2, +3, ...), zero (0), and the opposites of the counting numbers (−1, −2, −3, ...)

Whose birthday was on the coldest day?

A. Mark each daily low temperature on an **integer** number line like the one below.



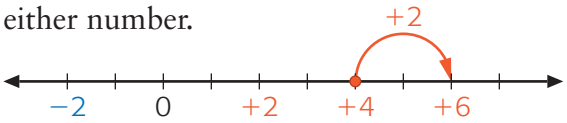
B. Whose birthday was on the warmest day? How do you know?

C. Whose birthday was on the coldest day? How do you know?

What Do You Think?

Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

1. When you add two numbers, the sum is always greater than either number.



2. It is 40 °C warmer inside than outside.

3. The integer −4 is halfway between −9 and +1.

6.1 An Integer Experiment

YOU WILL NEED
- a coin

GOAL
Add positive and negative integers.

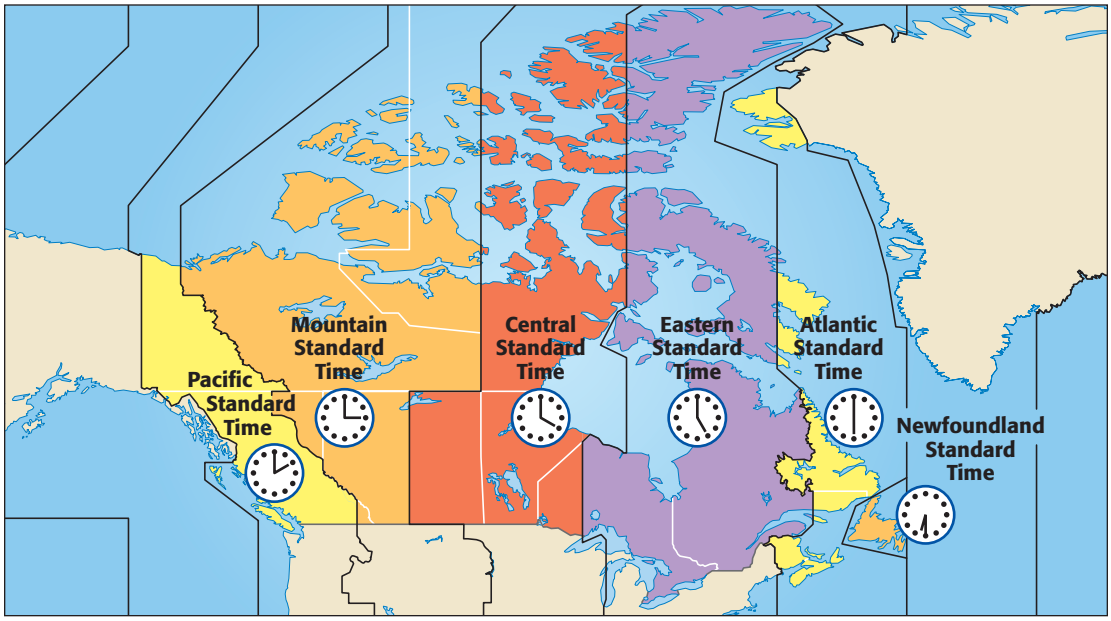
EXPLORE the Math

Nestor and Gail are playing a game. Nestor tosses a coin 20 times.
For each Head that Nestor tosses, Gail records +1.
For each Tail that Nestor tosses, Gail records −1.
Gail adds the positive and negative integers.
If the total is from −2 to +2, Nestor wins.
Otherwise, Gail wins.

What outcomes result in a win for Nestor?

CURIOUS MATH

Time Zones



Canada is divided into six time zones. For the five mainland zones, if you travel east, you add +1 hour to your watch each time you enter a new time zone. If you travel west, you add −1 hour to your watch each time you go to a new time zone.

1. What is the time in Vancouver when it is noon in Halifax? Explain how you added integers to solve the problem.
2. What is the time in each of the other time zones when it is noon in your time zone? Explain.
3. The clocks show the time in each city when it is 7:00 in Edmonton. How many hours behind or ahead of Edmonton is each city? Explain how you added integers to solve the problem.

4. Survey your classmates. In what other time zones do their relatives or friends live? What is the time in each of those time zones when it is noon in your time zone?

6.2 Adding Integers Using the Zero Principle

YOU WILL NEED
- red and blue counters
- a number line

GOAL

Use the zero principle to add integers.

LEARN ABOUT the Math

Nolan's favourite hockey player is Jordin Kudluk Tootoo, an Inuit who was born in Churchill, Manitoba.

Nolan recorded Jordin's +/− score over several games. If Jordin's team scored a goal while Jordin was on the ice, Nolan recorded +1 point. If the other team scored a goal while Jordin was on the ice, Nolan recorded −1 point. Here is Nolan's table for 11 goals.

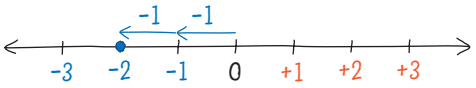
Goal	1	2	3	4	5	6	7	8	9	10	11
Result (+1) or (−1)	−1	−1	+1	+1	+1	−1	+1	−1	+1	−1	+1

How can you calculate Jordin's +/− score?

Example 1 | Using a number line

Calculate Jordin's +/− score using a number line.

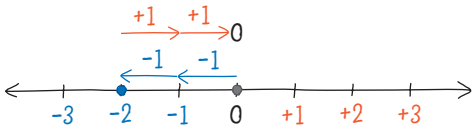
Nolan's Solution



(−1) + (−1) = (−2)

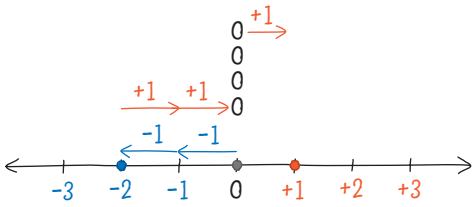
I added the first two integers in my table.
To represent −1, I drew an arrow pointing to the left.

I recorded the result using symbols.



$(-1) + (-1) = (-2)$

$(-2) + (+1) + (+1) = 0$



$0 + 0 + 0 + 0 + (+1) = (+1)$

I added the next two integers.
To represent +1, I drew an arrow pointing to the right.
This brought me back to 0.

The next two scores were +1 and −1. I knew they would bring me back to 0, so I just recorded the 0. There were two more pairs of 0s, which I recorded. Then I added the last integer.

Jordin's +/− score is +1.

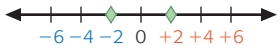
Example 2 | Modelling a sum with counters

Calculate Jordin's +/− score using counters.

Nayana's Solution

opposite integers

two integers that are the same distance from 0 on a number line; for example, +2 and −2 are opposite integers



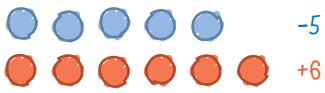
zero principle

the sum of two opposite integers is 0; for example,

(● ●) + (● ●) = 0

(−2) + (+2) = 0

I modelled the results. I used blue counters to represent −1s and red counters to represent +1s.



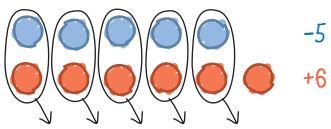
$(-5) + (+6)$

I put the blue counters in one row and the red counters in another row. The total is the sum of the counters.

$(+1) + (-1) = 0$

I paired a +1 with a −1 because they are **opposite integers.**

By the **zero principle,** the sum of +1 and −1 is 0.



$(-5) + (+6) = (+1)$

I was able to pair all the counters, except one. One red counter, or +1, was left over.

Jordin's +/− score is +1.

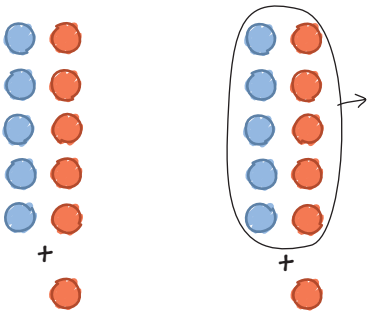
Communication | *Tip*

When you write the sign of an integer (+, −), use brackets so you do not confuse it with the operation signs (+ for addition or − for subtraction). For example, write the addition of +2 and −5 as (+2) + (−5). This is read as "positive two plus negative five."

Example 3 | Renaming an integer

Calculate Jordin's +/− score by renaming.

Liam's Solution



I used counters.
I renamed +6 as (+5) + (+1).

(−5) + (+5) = 0
(−5) + (+6) = (+1)

One red counter is left,
so Jordin's +/− score is +1.

Since −5 and +5 are opposite integers, their sum is 0.

Reflecting

A. How are Nayana's solution and Liam's solution alike? How are they different?

B. Nolan added each integer in order. How might he have paired more of the numbers to make the addition easier?

C. How do you think Liam knew that he could rename +6 as (+5) + (+1)?

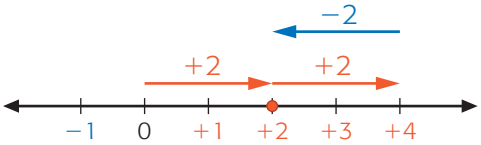
WORK WITH the Math

Example 4 | Adding integers

Calculate (+4) + (−2).

Solution A

Rename +4 as (+2) + (+2).



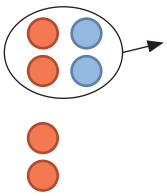
By the zero principle, (+2) + (−2) = 0.

So, (+4) + (−2) = (+2) + (+2) + (−2)
= (+2)

Solution B

Use counters to represent the integers.

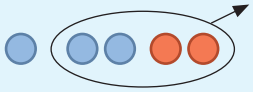
Group the counters to make opposite integers.



(+4) + (−2) = (+2)

A Checking

1. Add the integers in each expression using counters. Then record the addition using symbols. The first one is done for you.

	Expression	Addition model	Recording
a)	(−3) + (+2)		(−3) + (+2) = (−1)
b)	(−4) + (+6)		
c)	(+5) + (−6)		
d)	(−5) + (+7)		
e)	(+2) + (−8)		
f)	(−1) + (−9)		

2. Calculate.

a) (+3) + (−3) = ■ **b)** (−7) + (+7) = ■

B Practising

3. Complete.

a) $(-3) + (-2) = \square$

b) $(+2) + (-2) = \square$

c) $(-4) + (+1) = \square$

d) $(+3) + (-5) = \square$

e) $(-2) + (-1) = \square$

f) $(-6) + (+7) = \square$

4. Hailey's +/− score was +2 in one game and −3 in another game. What was her total +/− score?

5. Explain why $(-25) + (+25) = 0$.

6. Each pattern is based on adding integers. Continue each pattern. Then write a rule for the pattern.

a) $0, -1, -2, -3, -4, \square, \square, \square$

b) $-3, -2, -1, 0, \square, \square, \square$

7. Replace each $\square$ with +1 or −1 to make each statement true.

a) $(+1) + \square + \square = (-1)$

b) $(-1) + \square + \square = (+1)$

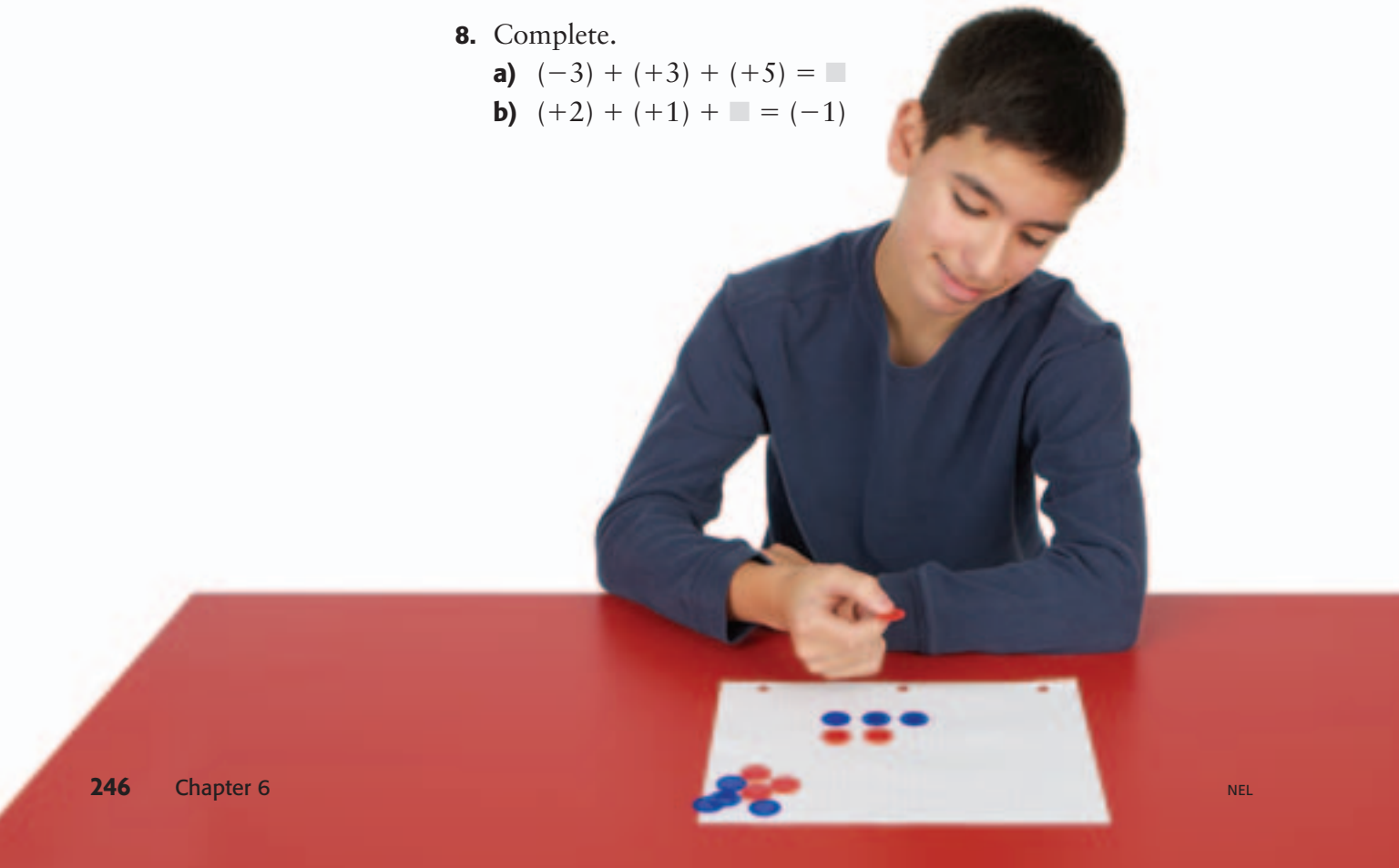
c) $(+1) + \square + \square + \square + \square = (-1)$

d) $(+1) + \square + \square + \square + (+1) = (-1)$

8. Complete.

a) $(-3) + (+3) + (+5) = \square$

b) $(+2) + (+1) + \square = (-1)$



9. Replace the ■ with $=$, $<$, or $>$ to make each statement true.

a) $(-1) + (-2) \blacksquare (-4)$ **d)** $(+5) + (-7) \blacksquare (-2)$

b) $(+2) + (-5) \blacksquare (-3)$ **e)** $(-2) + (-4) \blacksquare (-5)$

c) $(-3) + (+6) \blacksquare (+2)$ **f)** $(-2) + (+1) \blacksquare 0$

10. Using $+1$ and -1 only, create an addition question that has each sum. Use at least four numbers for each question.

a) $+3$ **b)** -2 **c)** 0 **d)** -1

11. Replace each ■ with an integer to make the equation true. Show three different solutions.

$\blacksquare + \blacksquare + \blacksquare = (-5)$

12. Explain why you cannot complete this equation using only $+1$s or -1s.

$(+1) + \blacksquare + \blacksquare + \blacksquare = (+1)$

$+1$	-6	-1
-4	-2	0
-3	$+2$	-5

$(-1) + 0 + (-5) = (-6)$

13. **a)** In a Magic Square, all rows, columns, and diagonals have the same sum. No number appears more than once. The Magic Square at the left uses integers from -6 to $+2$. Show that the rows, columns, and diagonals all have the same sum. The sum of the third column is shown.

b) Create a Magic Square that uses the integers from -10 to -2.

14. **a)** Add all the integers from -10 to $+10$. What pattern can you use to calculate the sum?

b) Add all the integers from -50 to $+50$, using the pattern in part (a).

15. Is each statement true or false? Explain your reasoning.

a) The sum of two positive integers is always positive.

b) The sum of two negative integers is always negative.

c) The sum of a negative integer and a positive integer is always positive.

6.3 Adding Integers that Are Far from Zero

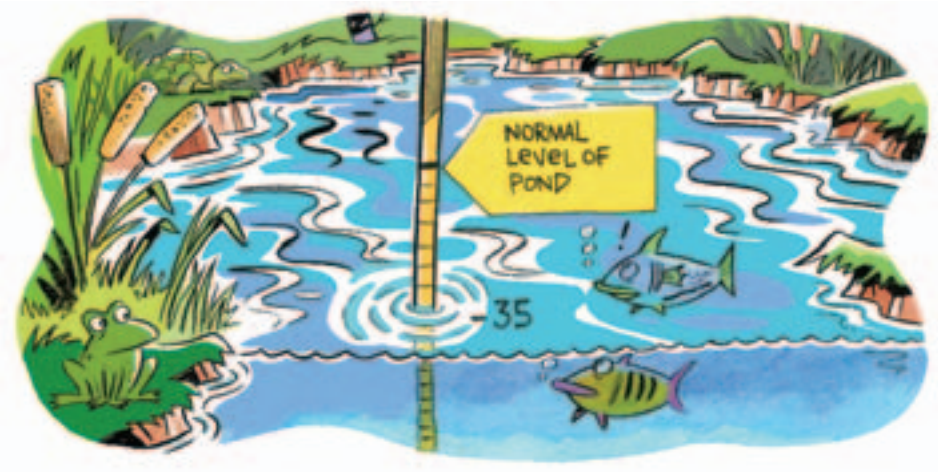
YOU WILL NEED
- a number line

GOAL

Add integers using number lines.

LEARN ABOUT the Math

Fiona is doing a project for a science fair. She is recording the rise and fall of the water level in a pond. One day, she had this result.



The next time Fiona measured the water level, it had risen by 40 cm.

What was the new reading on the rod?

A. Why can you solve the problem by calculating $(-35) + (+40)$?

B. Mark -35 on a number line. How can you represent adding $+40$ to -35?

C. Why might you want to first add $+35$ instead of $+40$?

D. Use your number line to calculate $(-35) + (+40)$. What was the new reading on the rod?

Reflecting

E. Another time, Fiona calculated $(+40) + (-35)$. What had happened to the water level in the pond?

F. Look at part A again. How can you tell, without calculating, whether the sum is positive or negative?

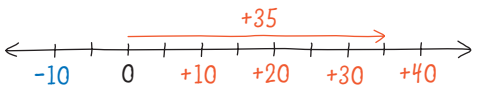
G. When is the sum of two integers negative? Use an example to help you explain.

WORK WITH the Math

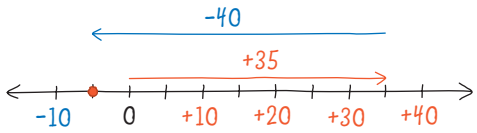
Example 1 | Adding integers on a number line

Add $+35$ and -40 on a number line.

Fiona's Solution



I thought of $+35$ as an increase from 0 to 35. I represented $+35$ with a red arrow going to the right.



I thought of -40 as a decrease. I represented -40 with a blue arrow starting at the tip of the first arrow and going to the left.

I ended up at -5. This makes sense, since -35 balances the $+35$ and there is still -5 left.

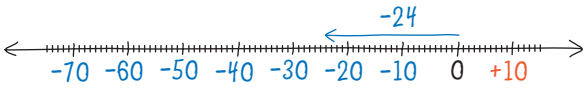
$(+35) + (-40) = (-5)$

The sum is -5.

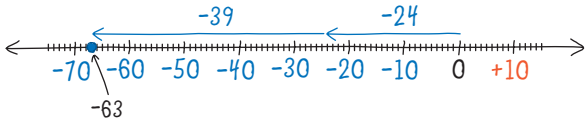
Example 2 | Adding negative numbers on a number line

Add $(-24) + (-39)$ on a number line.

Liam's Solution



I thought of -24 as a decrease from 0 to -24. I represented -24 with a blue arrow going to the left.



I thought of -39 as another decrease. I represented -39 with an arrow starting at the tip of the first arrow and going to the left.

$(-24) + (-39) = (-63)$

The sum is -63.

A Checking

1. Use a number line to model $(-27) + (+34)$.

a) Where does the first arrow start?

b) Where does the first arrow end?

c) Where does the second arrow start?

d) Where does the second arrow end?

e) What is the sum?

2. Predict whether each sum will be positive or negative. Then calculate the sum. Show what you did.

a) $(-50) + (-20)$

b) $(-50) + (+20)$

c) $(-20) + (+50)$

d) $(-20) + (-50)$

B Practising

3. Calculate.

a) $(+5) + (+3)$

b) $(-5) + (-3)$

c) $(-60) + (+20)$

d) $(-10) + (-15)$

e) $(-15) + (+10)$

f) $(+100) + (-80)$

4. How much greater is the second sum than the first sum? Show your work.

a) $(-25) + (+38)$ and $(-15) + (+38)$

b) $(+125) + (-52)$ and $(+125) + (-32)$

−1		+3
		−4

5. In the Magic Square at the left, every row, column, and diagonal adds to 0. Copy and complete this Magic Square.

6. Order the players from highest to lowest +/− score.

Player	Goals for (+)	Goals against (−)
Heidi	110	94
Rana	103	89
Meagan	99	108
Sonya	105	97
Indu	101	102

7. Copy and complete this table.

	Starting temperature (°C)	Temperature change (°C)	Final temperature (°C)
a)	−5	+1	
b)	−10	−6	
c)	0		−8
d)		−5	0
e)	+7		−2
f)		−10	+8

8. Why can you calculate $(-20) + (+8)$ by calculating $20 - 8$ and taking the opposite of your result?

9. Explain why you can calculate $(+20) + (-8)$ by calculating $20 - 8$.

10. How can you calculate $(-20) + (-8)$ without counters or a number line?

6.4 Integer Addition Strategies

YOU WILL NEED
- a number line
- red and blue counters

GOAL

Develop personal addition strategies for integers.

LEARN ABOUT the Math

Fiona is following the launch of a space shuttle on the NASA website. The countdown clock shows the number of hours before launch with negative integers and the number of hours after launch with positive integers.

It is now 31 h before launch, or −31. Exactly 12 h ago, the ground crew started an equipment check. In 12 h, the astronauts will begin their final preparations. Fiona wonders what the clock will show 12 h from now, and what it showed 12 h before.

What are the countdown integers?

A. Fiona calculated (−31) + (+12). What problem did she solve?

B. Calculate (−31) + (+12) using counters. Draw pictures and describe each step.

C. Calculate (−31) + (+12) on a number line. Draw pictures and describe each step.

D. Fiona calculated (−31) + (−12). What problem did she solve?

E. Calculate (−31) + (−12) using counters or a number line. Draw pictures and describe each step.

F. What will the integers be in 12 h, and what were they 12 h ago?

Reflecting

G. In part A, how could you tell that the sum would be greater than −31?

H. In part D, how could you tell that the sum would be less than −31?

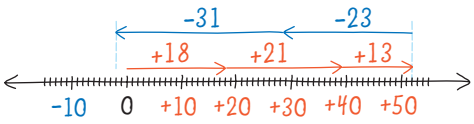
I. Suppose that you want to add an integer to −31. How can you tell whether the answer will be positive, negative, or zero?

WORK WITH the Math

Example 1 | Adding integers on a number line

Calculate (+18) + (−23) + (−31) + (+21) + (+13).

Fiona's Solution: Adding positives and negatives



(+18) + (+21) + (+13) = (+52)

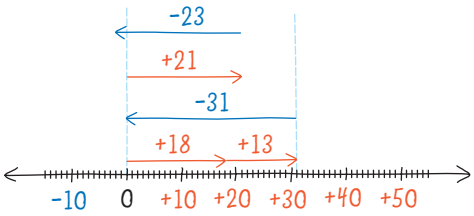
(−23) + (−31) = (−54)

(+52) + (−54) = (−2)

I grouped the positive integers and the negative integers on a number line.

The sum of the positive integers is +52.
The sum of the negative integers is −54.
The sum is −2.

Nayana's Solution: Using zero pairs



I noticed that (+18) + (+13) = (+31).

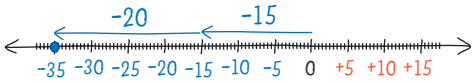
+31 and −31 are a zero pair, so +21 and −23 are left.

I added (+21) + (−23) on a number line.
The sum is −2.

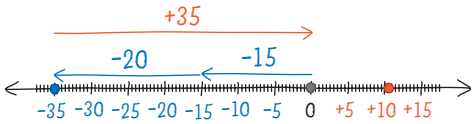
Example 2 | Adding positive and negative integers

Calculate (+35) + (+11) + (−15) + (−20).

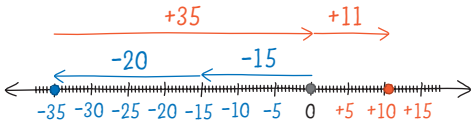
Megan's Solution: Using zero pairs



I added −15 and −20 on a number line.
The sum is −35.



I added +35 to get 0.

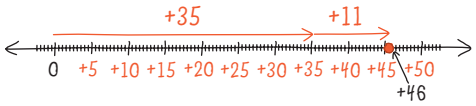


+11 is left. I added +11 to 0.

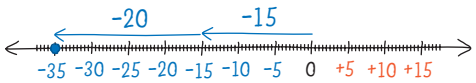
The sum is +11.

(−15) + (−20) = (−35)
(−35) + (+35) = 0
0 + (+11) = (+11)

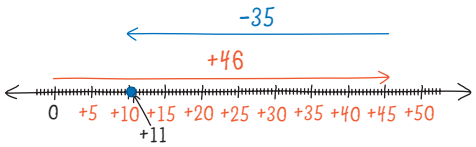
Nolan's Solution: Adding positives, then negatives



I added the two positive integers.



I added the two negative integers.



I added +46 and −35.

The sum is +11.

(+46) + (−35) = (+11)

A Checking

1. Calculate.

a) $(-40) + (+55) + (+5) + (-40) + (-10)$

b) $(-13) + (+8) + (-12) + (+10) + (+9)$

B Practising

2. Explain why all of these sums are the same.

A. $(-5) + (-2) + (-3) + (+5)$

B. $(-5) + (+5) + (-2) + (-3)$

C. $(-2) + (-3) + (+5) + (-5)$

D. $(+5) + (-2) + (-3) + (-5)$

3. Troy had a +/− score of +11. The following table shows how his score changed over seven games. What is his score now?

Game	start	1	2	3	4	5	6	7
+/− score		−1	+4	−3	−2	+5	0	+1
Total	+11							

4. Look at the expression $(+4) + (-3) + (+1) + (+6) + (-2)$.

a) Add the integers in order.

b) Group and add the positive integers. Then group and add the negative integers. Add the two sums.

c) Why do you get the same answer in parts (a) and (b)?

5. Calculate.

a) $(-12) + (+2) + (-5)$

b) $(+23) + (-14) + (-7)$

c) $(-18) + (+5) + (+18)$

d) $(-10) + (-15) + (+15) + (+20)$

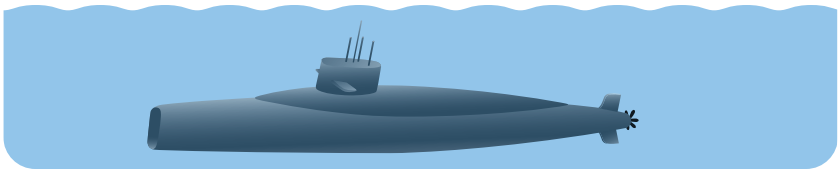
6. Replace each ■ with a different two-digit integer to make the equation true. Write two solutions.

■ + ■ + ■ + ■ + ■ = (+4)

Reading Strategy

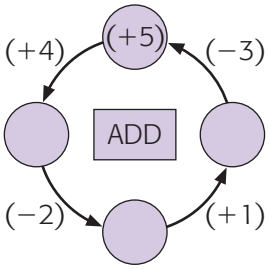
What is the useful information in this problem?
Why is it useful?

7. The depth of a submarine is shown at 10:00. How deep was it at 15:00?



Time	10:00	11:00	12:00	13:00	14:00	15:00
Change in depth (m)		−53	−31	+18	−64	+85
Depth (m)	−300					

8. a) Copy the diagram shown below. Start with +5 in the top circle. Fill in the other circles by following the arrows and adding the indicated integers.



b) Why is the final sum +5 after you finish the last addition?

c) Why is the final sum +5 if you add in both a clockwise direction and a counterclockwise direction?

d) Copy the diagram again. Replace the numbers on the arrows with four different two-digit integers, so that you still end up with a sum of +5.

9. Describe two ways to add these integers. State which way you prefer and explain why.
(+22) + (−4) + (+12) + (−10) + (+8) + (−6)

Mid-Chapter Review

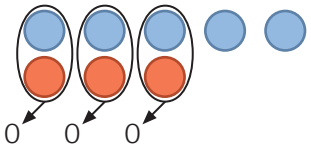
Frequently Asked Questions

Q: **What is the zero principle?**

A: The zero principle means that the sum of two opposite integers is 0. For example, $(+3) + (-3) = 0$.

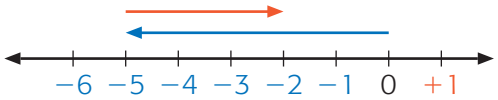
Q: **How do you add integers, such as $(-5) + (+3)$?**

A1: You can use counters. You can pair red and blue counters and use the zero principle. Each pair of red $(+1)$ and blue (-1) counters has a sum of 0. Then remove the zero pairs. The counters that remain give the sum. This model shows that $(-5) + (+3) = (-2)$.



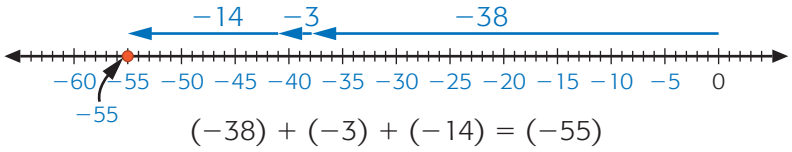
$(+1) + (-1) = 0$

A2: You can use a number line. Represent the first integer with an arrow that starts at 0. Represent the second integer with an arrow that starts at the end of the first arrow. An arrow points to the left if it represents a negative integer and to the right if it represents a positive integer. The sum is the end point of the second arrow. This model shows that $(-5) + (+3) = (-2)$.

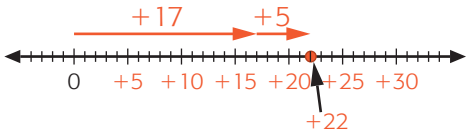


Q: **How do you add more than two integers, such as $(-38) + (+17) + (-3) + (-14) + (+5)$?**

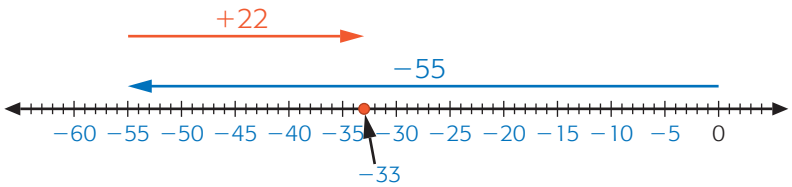
A1: You can add the positive and negative integers separately, and then calculate the total sum. For example,



$(-38) + (-3) + (-14) = (-55)$

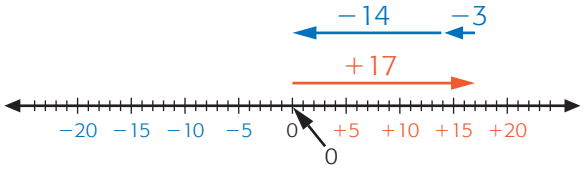


$(+17) + (+5) = (+22)$

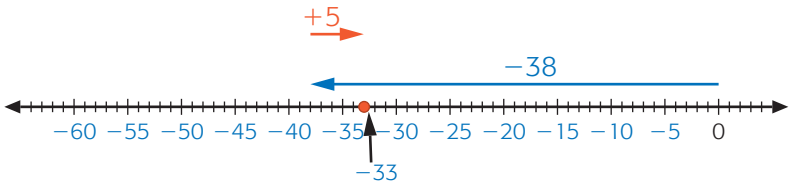


$(-55) + (+22) = (-33)$

A2: You can look for zero pairs and remove them. Then you add the remaining integers on a number line or with counters. For example, in $(-38) + (+17) + (-3) + (-14) + (+5)$, notice that $(+17)$ makes a zero pair with $(-3) + (-14)$.



This leaves $(-38) + (+5)$, so the sum is -33.



$(-38) + (+5) = (-33)$

Practice

Lesson 6.2

1. Draw a picture to represent each sum.

a) $(+5) + (-2)$
b) $(-2) + (-5)$
c) $(-4) + (+5)$
d) $(+3) + (-4)$
e) $(-2) + (+7)$
f) $(-1) + (-3) + (+4)$

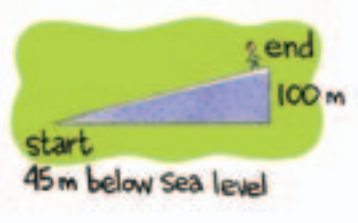
2. Complete each equation.

a) $(-8) + \square = (-5)$
b) $(+2) + \square = 0$
c) $(-5) + \square + (+7) = (+12)$
d) $(+6) + \square + (-4) = (+10)$

3. Think about adding two integers.

a) What must be true about the integers for the sum to be positive?
b) What must be true about the integers for the sum to be negative?

Lesson 6.3



4. Anthony hiked uphill from a valley that was 45 m below sea level. After an hour, he was 100 m higher than where he started. Using integers, determine how high he was above sea level. Show what you did.

5. Calculate.

a) $(+30) + (-20)$
b) $(-150) + (+50)$
c) $(-110) + (-20)$
d) $(+20) + (-40)$

Lesson 6.4

6. Calculate.

a) $(+11) + (-26) + (-15)$
b) $(-33) + (-20) + (+12)$
c) $(-50) + (+23) + (-17)$
d) $(-40) + (+20) + (-14)$

7. Calculate.
$(+34) + (+17) + (-20) + (-15) + (-2) + (+18)$

8. Gillian walks up and down a staircase. She starts on the fifth step and walks as follows:

- up 2 steps
- down 3 steps
- up 4 steps
- down 5 steps

On what step does she finish?

6.5 Subtracting Integers Using Counters

YOU WILL NEED

- red and blue counters

GOAL

Develop a counter model for subtracting integers.

LEARN ABOUT the Math

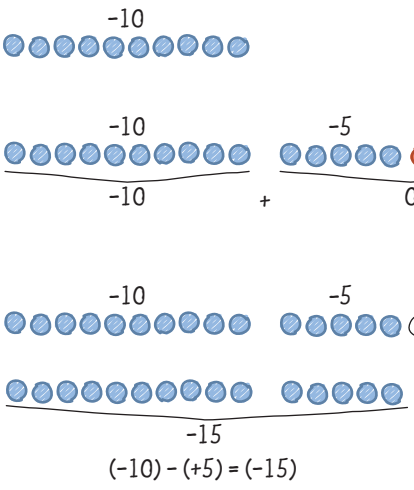
Ashley, Nick, and Gail have electronic gift cards to buy items online. They can add money to their cards when the balance is low or negative. Money is subtracted from their cards when they buy an item.

How can you subtract integers using counters?

Example 1 | Subtracting a positive integer

Ashley's card had a balance of −$10. Then she spent $5. What is the balance now?

Ashley's Solution



I had to calculate (−10) − (+5).
I used blue counters to represent −10.

I know that −10 + 0 is still −10.
I wanted to subtract +5, so I needed five red counters. I used the zero principle to get more blue and red counters.

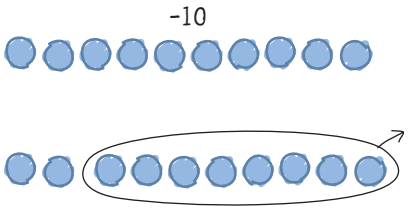
I subtracted 5 red counters, or +5.

The balance on my card is now −$15. It makes sense that the balance is less than −$10, because now I should owe more.

Example 2 | Subtracting a negative integer

Nick ordered a book worth $8. The balance on his card became −$10. Then Nick decided to cancel the order. What is the balance now?

Nick's Solution



When I cancelled the order, I had to subtract −$8 from my balance of −$10 to get my new balance.
I calculated (−10) − (−8).
I used blue counters to represent −10.

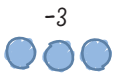
I had enough blue counters to subtract −8.

My balance is now −$2. It makes sense that the answer is more than −$10, because now I should owe less.

Example 3 | Subtracting integers by adding

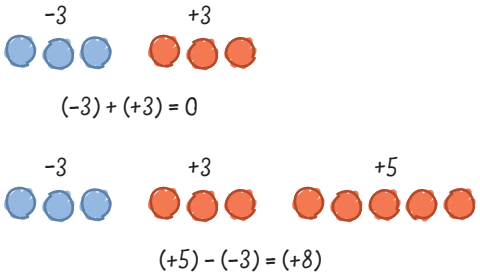
Calculate +5 − (−3).

Gail's Solution



You can subtract whole numbers by adding on to the second number.
5 − 3 = ■ means "What number can you add to 3 to get 5?"
So I can rewrite the equation as 3 + ■ = 5.
Subtracting with integers is the same. (+5) − (−3) = ■ means "What number can you add to −3 to get +5?"
I can rewrite the equation as (−3) + ■ = (+5).

I used 3 blue counters to represent −3.



I had to get a result of +5, or 5 red counters, so I added red counters.
First I added 3 red counters, or +3, to get 0.

I added another 5 red counters to get +5.
I added 8 red counters in all, or +8, to −3.
The answer is +8.

Reflecting

A. How could Ashley and Nick add to check the new balances they calculated?

B. Suppose that you subtract −3 from an integer. Will the difference be less than or greater than the integer? Use an example to help you explain.

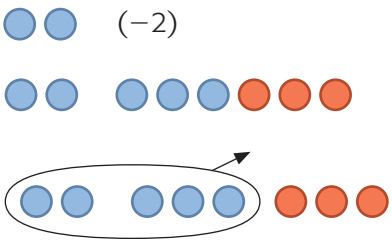
C. How is subtracting −5 from an integer like adding +5 to the same integer? Use an example to help you explain.

WORK WITH the Math

Example 4 | Subtracting integers with the same sign

Calculate (−2) − (−5) using counters.

Solution



Start with 2 blue counters.

Add 0 by adding 3 blue and 3 red counters.

Subtract 5 blue counters.

Three red counters are left.

(−2) − (−5) = (+3)

A Checking

1. Calculate using counters. For which calculations did you need to add red and blue counters that equalled 0?

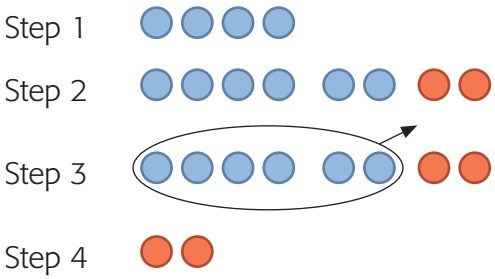
a) $(-4) - (+2)$
b) $(+3) - (+2)$
c) $(+3) - (-2)$
d) $(-3) - (-2)$

B Practising

2. a) Calculate $(-4) - (-1)$ using counters.
b) Why didn't you need to add red and blue counters that equalled 0?

3. a) Calculate $(-1) - (-4)$ using counters.
b) Why did you need to add red and blue counters that equalled 0?

4. Rosa solved a subtraction problem as shown. What problem did she solve?



5. Are $(+6) - (-4)$ and $(-4) - (+6)$ equal? Use counters to help you explain.

6. Which expressions have the same result? Use counters to help you explain.

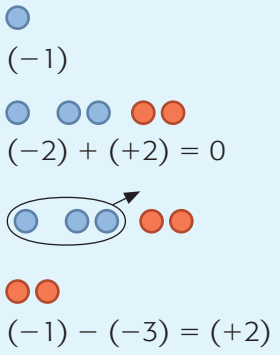
A. $(+3) - (+4)$
B. $(-4) - (-3)$
C. $(+9) - (+5)$
D. $(-4) - (-9)$
E. $(-4) - (+3)$
F. $(+4) - (-3)$

7. Complete each pattern. What does the pattern show about subtracting integers?

a) $(+5) - (+4) = (+1)$
$(+5) - (+3) = \square$
$(+5) - (+2) = \square$
$(+5) - (+1) = \square$

b) $(-5) - (+4) = (-9)$
$(-5) - (+3) = \square$
$(-5) - (+2) = \square$
$(-5) - (+1) = \square$

8. Subtract the second integer from the first integer. Model the subtraction with counters and record the result. The first one is done for you.

	First integer	Second integer	Model
a)	−1	−3	 (−1) (−2) + (+2) = 0 (−1) − (−3) = (+2)
b)	−4	+5	
c)	+6	−10	
d)	0	−10	
e)	+7	+10	
f)	−1	−8	

9. The following table shows the balances on some electronic gift cards. Copy and complete the table.

	Person	Day 1 balance ($)	Day 2 balance ($)	Change in the balance ($)
a)	Ming	−5	−1	
b)	Kaitlyn	+5	+10	
c)	Omar	−10	+6	
d)	Anthony	−10		−8
e)	Braydon		−5	+10
f)	Tynessa		+7	+2

10. Calculate.

a) $(+4) + (+2) - (+3)$

b) $(-4) + (-3) - (-2)$

c) $(+3) - (-8) + (-10)$

d) $(-2) - (-6) + (+3)$

11. Is each statement true or false? Explain your thinking using examples.

a) When you subtract a negative integer from another negative integer, the result is always negative.

b) When you subtract a positive integer from another positive integer, the result is always positive.

c) When you subtract a negative integer from a positive integer, the result is always positive.

d) When you subtract a positive integer from a negative integer, the result is always positive.

e) When you subtract one integer from another integer, the result always has the same sign as the greater integer.

12. The opposite of $+3$ is -3.

a) How do you know that $(-2) - (+3) = (-2) + (-3)$?

b) Is subtracting an integer always the same as adding its opposite? Use counters to explain your answer.

MATH GAME

Integro

If you are using a standard deck of cards, aces count as 1, numbered cards count as their face values, and jokers count as 0. Red cards are positive, and black cards are negative.

Number of players: 2 or 4

YOU WILL NEED

- integer cards numbered −10 to +10 (two of each) OR a standard deck of cards, including two jokers, with face cards removed

How to Play

1. If there are four players, remove the jokers. Shuffle the cards. Deal the cards equally to all the players.
2. In a round, each player places one card face up on the table.
3. The first player to call out the sum of the cards wins all the cards in the turn. These cards go into the player's bank pile.
4. If there is a tie, the tied players play additional rounds until one of them wins.
5. When a player runs out of cards, the player shuffles his or her bank pile and continues playing. If the player's bank is empty, the player is out of the game.
6. The game ends when one player has won all the cards.

6.6 Subtracting Integers Using Number Lines

YOU WILL NEED

- a number line

GOAL

Calculate the difference between integers using a number line.

LEARN ABOUT the Math

Matthew, Julie, and Sarah researched the record extreme temperatures of the capital cities in western and northern Canada.

Capital city	Lowest recorded temperature (°C)	Highest recorded temperature (°C)	Difference (°C)
Winnipeg	−45	+41	
Iqaluit	−46	+25	
Yellowknife	−51	+33	
Whitehorse	−52	+34	
Regina	−50	+43	
Edmonton	−48	+35	
Victoria	−16	+36	

They wanted to know how much higher the highest temperature was than the lowest temperature for each city.

What is the difference between the highest temperature and the lowest temperature for each city?

A. Suppose that the lowest Winnipeg temperature was +2. How would you express the temperature difference on a number line?

B. Express the difference between Winnipeg's actual temperatures as a subtraction.

C. Calculate the difference between Winnipeg's temperatures on a number line.

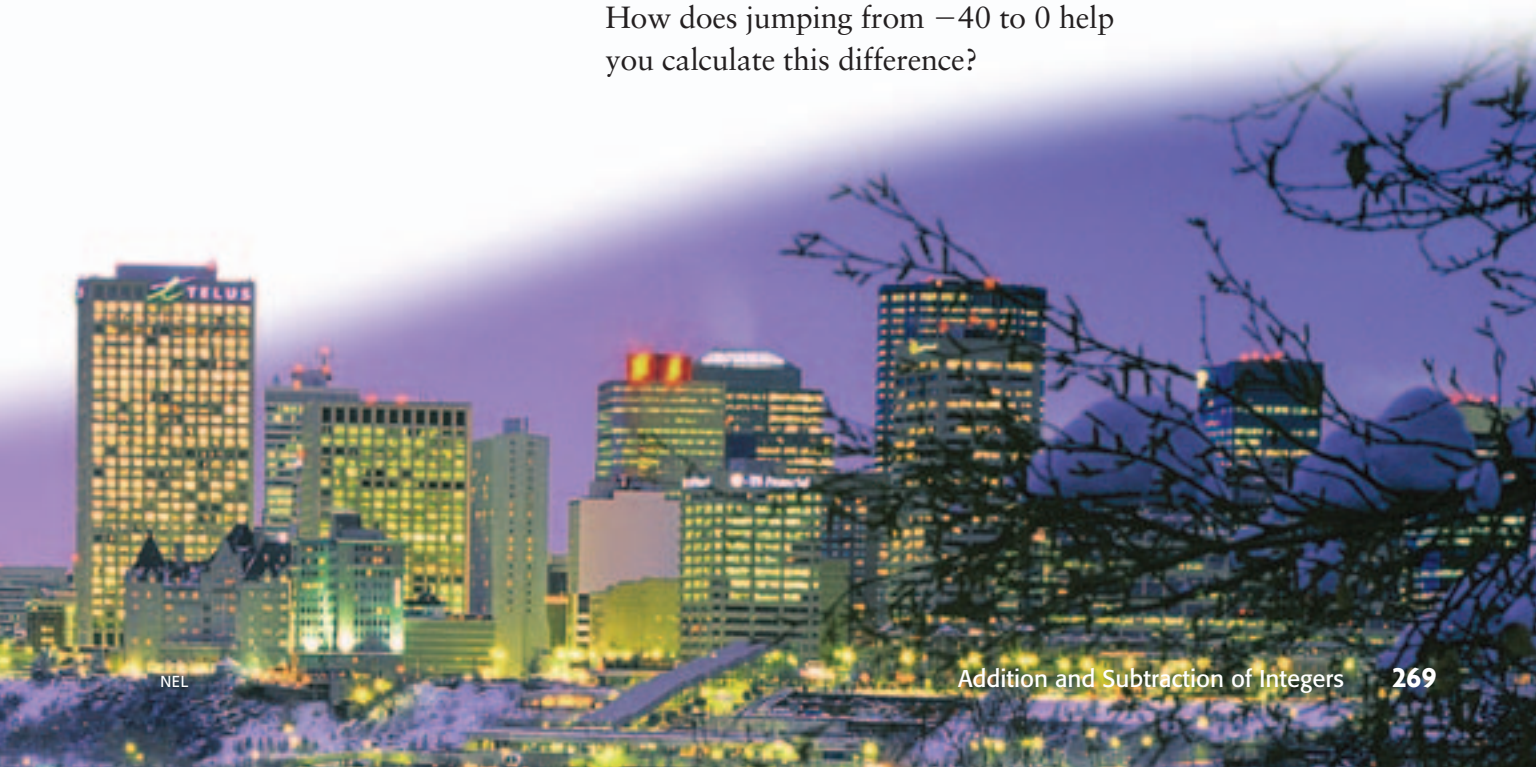
D. Calculate the difference between the highest and lowest temperatures for each of the other cities on a number line. Record your subtraction for each difference.

Reflecting

E. What temperature problem would you be solving if you calculated $(-45) - (+41)$? Use a number line to help you explain.

F. How can you tell, without calculating, that the answer for part C is a positive integer?

G. Suppose that you use a number line to calculate $(+35) - (-40)$. How does jumping from -40 to 0 help you calculate this difference?

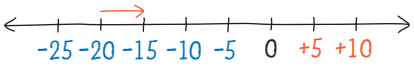


WORK WITH the Math

Example 1 | Subtracting integers on a number line

Calculate (−15) − (−20) on a number line.

Julie's Solution



I wanted to calculate the difference between −20 and −15.

I started at −20 and went to −15.

The arrow is 5 units long.

The arrow points to the right, so I recorded the difference as a positive integer.

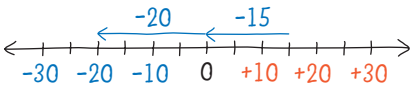
(−15) − (−20) = (+5)

The difference is +5.
This makes sense, because (−20) + (+5) = (−15).

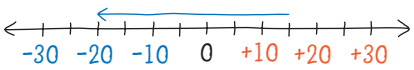
Example 2 | Subtracting integers on a number line

Calculate (−20) − (+15) on a number line.

Matthew's Solution



I wanted to calculate the difference between +15 and −20.
I started at +15.
It was 15 units from +15 to 0
and 20 more units to −20.



The arrow is 35 units long.

The arrow points to the left, so I recorded the difference as a negative integer.

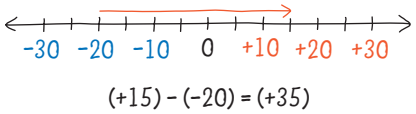
(−20) − (+15) = (−35)

The difference is −35.
This makes sense, because (+15) + (−35) = (−20).

Example 3 | Subtracting integers on a number line

Calculate (+15) − (−20) on a number line.

Sarah's Solution



(+15) − (−20) = (+35)

I wanted to calculate the difference between −20 and +15.

I started at −20 and went to +15. It was 20 units from −20 to 0, and 15 more units to +15. The arrow is 35 units long. The arrow points to the right, so I recorded the difference as a positive integer.

The difference is +35.
This makes sense, because (−20) + (+35) = (+15).

A Checking

1. **a)** Calculate (−35) − (+40) on a number line.
b) What is the starting point of the arrow?
c) What is the end point of the arrow?

2. Calculate.
a) (−12) − (−20)
b) (+31) − (+32)

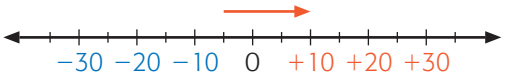
B Practising

3. **a)** Calculate (+36) − (−34) = ■ and (−34) − (+36) = ▲ on a number line.
b) Explain why ■ and ▲ are opposite integers.

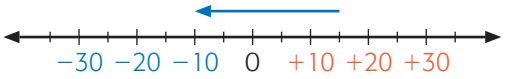
4. Calculate.
a) (−20) − (−40)
b) (+30) − (+70)
c) (−23) − (−21)
d) (+35) − (+32)
e) (+10) − (−10)
f) (−20) − (−20)

5. Record the subtraction that each model represents.

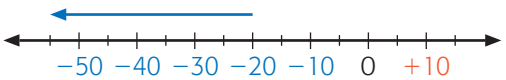
a)



b)



c)



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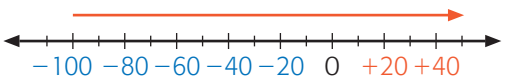


6. The difference between two integers is -5. What does this tell you about the positions of the integers on a number line?

7. Determine the unknown value.

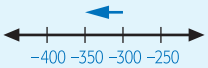
a) $\square - (-4) = -30$

b) $(-12) - \square = -19$

c) $\square - (+7) = -32$

d) $(+8) - \square = -15$

8. The following table shows the changing balances in some bank accounts. Copy and complete the table.

	Starting balance ($)	Final balance ($)	Model	Change in value ($)
a)	−300	−350		(−350) − (−300) = (−50)
b)	+200	−150		
c)	+150	+20		
d)	−595	+105		
e)	−1005	−950		
f)	+537	−111		

9. Which of these expressions has the greatest result?

A. $(+40) + (+20) - (+30)$

B. $(+37) - (-85) + (-10)$

C. $(-100) - (-510) + (-520)$

D. $(+25) + (-40) - (-135)$

10. **a)** Explain why $(+15) - (-9) = (+15) + (+9)$.

b) Can you always add the opposite to subtract an integer? Use an example to help you explain.

6.7 Solve Problems by Working Backward

YOU WILL NEED

- a number line
- red and blue counters

GOAL

Solve problems using the strategy of working backward.

LEARN ABOUT the Math

Nayana showed a number trick to Nestor.
She told him to follow these steps:

1. Think of a number.	→	**2.** Add +1.	→	**3.** Subtract −3.	→	**4.** Add −5.	→	**5.** Name the result.

Nestor said that his result was −2.
Nayana said, "I think your starting number was −1."

How did Nayana know Nestor's starting number?

1 Understand the Problem

Nestor wants to know how Nayana determined his number from his result.

2 Make a Plan

Nestor realizes that he needs to start with the result and work backward through the steps to figure out the original number.

3 Carry Out the Plan

Nestor completes the original steps in order.
Then he works backward from the result.

Original Steps

1. Think of a number.	→	**2.** Add +1.	→	**3.** Subtract −3.	→	**4.** Add −5.	→	**5.** Name the result.
?		+(+1)		−(−3)		+(−5)		= (−2)

Working Backward

5. The result is -2.	**4.** Subtract -5.	**3.** Add -3.	**2.** Subtract $+1$.	**1.** The number I thought of is -1.
(-2)	$-(-5)$	$+(-3)$	$-(+1)$	$=(-1)$

It works!

Reflecting

A. How does working backward help Nestor solve Nayana's number trick?

WORK WITH the Math

Example | Working backward

Bill played three rounds in a golf tournament. His second-round score was 6 lower than his first-round score. His third-round score was 2 higher than his second-round score. His score for the third round was -3. What was his first-round score?

Nayana's Solution

1 Understand the Problem

I need to determine the first-round score. I already have the score for the third round. I also know how he did in the second round.

2 Make a Plan

I will draw boxes to show each step. Then I will work backward.

3 Carry Out the Plan

Each box represents one step.

First-round score → Add -6. → Second-round score → Add $+2$. → Third-round score is -3.

I can work backward from the result.

Third-round score is -3. → Subtract $+2$. → Second-round score → Subtract -6. → First-round score

$(-3) - (+2) = (-5)$
His second-round score was -5.

$(-5) - (-6) = (+1)$
His first round score was $+1$.

A Checking

1. Try Nayana's trick using another number. Is there a quick way to figure out the original number? If so, explain how it works.

2. John did this number trick.

- Think of a number.
- Add +2.
- Subtract −1.
- Add −2.
- The result is +4.

What was the original number? State the steps, in order, that you used.

B Practising

3. Jane did this calculation.

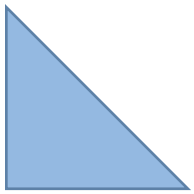
- Add −12.
- Subtract −9.
- Add +8.
- Subtract −2.
- The result is +5.

What was the original number? State the steps, in order, that you used.

4. Make up a number trick that gives you the original number as the result. Your trick must have at least four steps. The last step must be subtract +3.

5. Lloyd is lifting weights over a nine-week training period. Every week, he lifts 2 kg more than he lifted the previous week. During the ninth week, he lifts 80 kg. How much was he lifting during his first week?

6. During a clothing sale, the price goes down by half each day an item is not sold. If an item costs $2.50 after eight days, what was the original price?

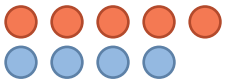


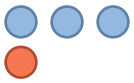
7. Ramona takes a shape and cuts away half of it five times. The triangle at the left is what remains. Draw the original shape.

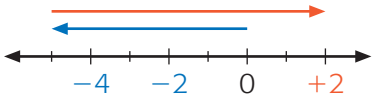
8. Make up a problem you can solve by working backward. Show how to solve it.

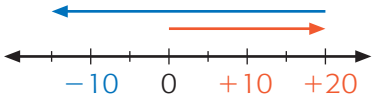
Chapter Self-Test

1. Calculate.

a) 

b) 

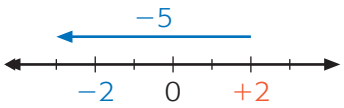
c) 

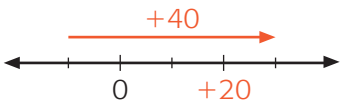
d) 

2. Calculate. Then explain what you did in each calculation.

a) $(-2) + (+3)$ **b)** $(+3) + (-2)$ **c)** $(-3) - (+4)$

3. Write the subtraction that each model represents.

a) 

b) 

4. Calculate.

a) $(+5) + (-8)$
b) $(-10) + (-3)$
c) $(-7) - (+2)$
d) $(-8) - (-4)$

5. On Monday morning, Sam had $50 in his wallet. The following table shows how this amount changed over the week. How much did Sam put into, or take out of, his wallet on Friday afternoon?

Day	start	Mon.	Tues.	Wed.	Thu.	Fri.
Gain/loss ($)		+30	−19	−25	+51	
Total ($)	50					20

6. **a)** How could you predict that $(-4) + (+12)$ is positive and 8 units to the right of 0?

b) How could you predict that $(-15) - (+23)$ is negative and 38 units to the left of 0?

What Do You Think Now?

Revisit What Do You Think? on page 239. How have your answers and explanations changed?

Chapter Review

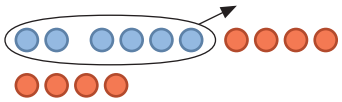
Frequently Asked Questions

Q: How do you subtract integers, such as $(-2) - (-6)$?

A1: You can use counters. If necessary, you can use the zero principle to add red and blue counters that equal 0. Then you remove the counters you need to. The remaining counters represent the answer. This model shows that $(-2) - (-6) = (+4)$.

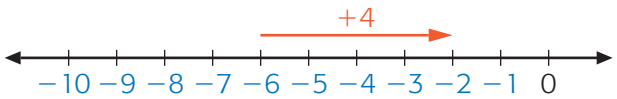
(-2)

$(-4) + (+4) = 0$

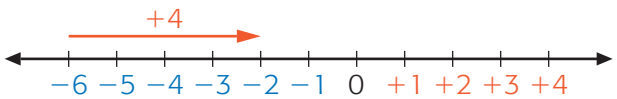


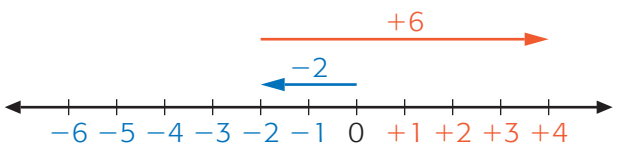
$(-2) - (-6) = (+4)$

A2: You can use a number line. Determine the difference between the second number and the first number. If the arrow from the second number points to the left, the answer is negative. If the arrow points to the right, the answer is positive. This model shows that $(-2) - (-6) = (+4)$.



A3: You can add the opposite. This model shows that $(-2) - (-6)$ has the same value as $(-2) + (+6)$, which is $+4$.





Practice

Lesson 6.2

1. Write three different addition questions that have a result of -2.

Lesson 6.4

2. On Monday morning, Polly had \$30 in her wallet. How much did Polly put into, or take out of, her wallet on Friday?

Day	start	Mon.	Tues.	Wed.	Thu.	Fri.
Gain/loss (\$)		+4	+5	−2	−10	
Total (\$)	30					17

3. Calculate.

a) $(-12) + (-6) + (-18)$ **b)** $(-37) + (-20) + (+12)$

Lesson 6.5

4. ▢ represents an integer. Which is greater, ▢ $-$ $(+1)$ or ▢ $+$ $(+1)$? Explain your reasoning.

5. Which expression has the greatest result? Which expression has the least result?

A. $(+4) - (+2)$
B. $(-7) - (+4)$
C. $(+6) - (-3)$
D. $(-3) - (-5)$

6. How is subtracting integers like adding integers? How is it different?

Lesson 6.6

7. Calculate.

a) $(+8) + (-3)$ **c)** $(-7) + (+6)$
b) $(-8) - (-3)$ **d)** $(+2) - (+4)$

8. One integer is 5 greater than another integer. Their sum is -13. What are the two integers?

Lesson 6.7

9. The temperature dropped 5 °C from midnight to noon. Then it rose 10 °C from noon to 10:00 p.m. It is now -25 °C. What was the temperature at midnight?

Chapter Task

Task | Checklist

- ✔ Do your clues involve both addition and subtraction of integers?
- ✔ Does at least one of your clues compare integers?
- ✔ Did you include a clue you did not need?
- ✔ Did you check to make sure that your clues give the correct integers?

Mystery Integer

Choose three integers. Make up a set of clues that will allow others to figure out which integers you chose. All of the clues must be necessary.

The clues must

- include both addition and subtraction of integers
- compare integers

For example, suppose that your integers are -8, $+17$, and -33. Here are three possible clues:

- If you subtract one integer from another, the result is $+25$.
- If you add all three integers, the result is less than -20.
- The sum of the least and greatest integers is -16.

What clues can you write to describe your three integers?

A. The three clues above do not give enough information for someone to figure out the integers. What clues can you add to give enough information?

B. Select three integers of your own, and make up some clues.