Chapter

5

Measurement

GOAL

You will be able to

- develop and apply a formula for the area of a parallelogram
- develop and apply a formula for the area of a triangle
- construct a circle with a given radius or diameter
- develop and apply formulas for the circumference and area of a circle

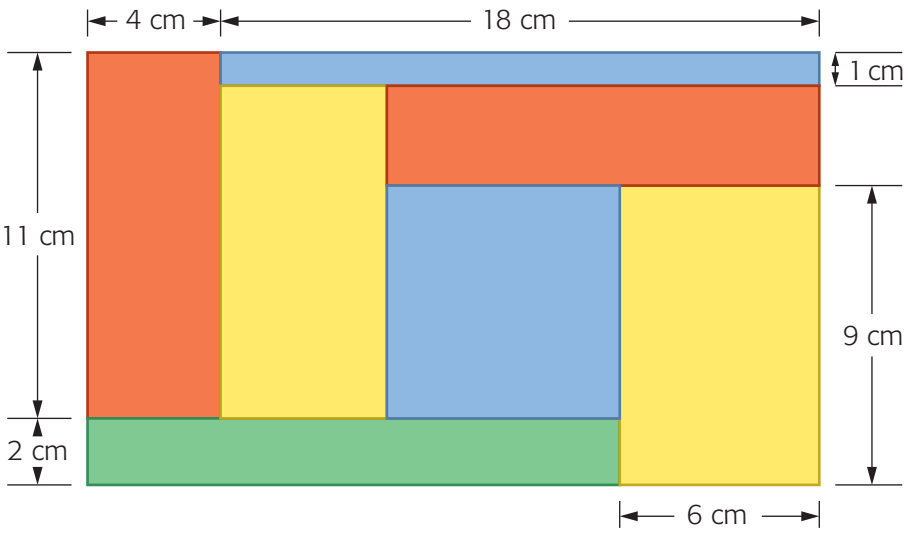
This window is about 50 cm wide. How can you estimate its area?

Getting Started

YOU WILL NEED
- a calculator

Creating Geometric Art

Argentinian artist Jesus Soto creates art using geometric shapes and fabrics. Nolan and Fiona were inspired by his work, and they decided to create their own art using coloured fabrics. First, they sketched a design.



How much of each coloured fabric will they need?

A. Use the given dimensions to determine the missing dimensions.

B. What is the area of the entire design, in centimetres?

C. What is the total area of the yellow fabric in the design?

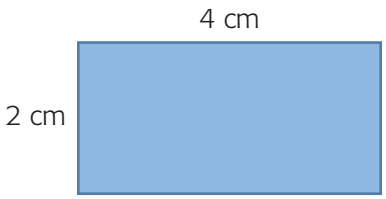
D. What percent of the design is green?

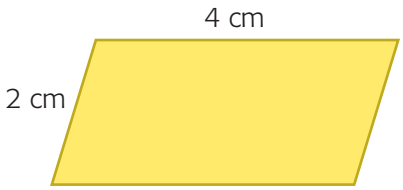
E. What is the ratio of each coloured fabric to the total area of the design?

What Do You Think?

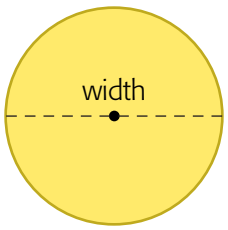
Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

1. A parallelogram and a rectangle that have the same side lengths also have the same area.





2. The distance around a circle is about twice as great as the width of the circle.



3. A triangle can be $\frac{1}{4}$ of, $\frac{1}{3}$ of, or $\frac{1}{2}$ of a parallelogram.

4. A circle and a square cannot have the same perimeter.

5.1 The Area of a Parallelogram

YOU WILL NEED

- a calculator
- pencil crayons
- 1 cm square dot paper
- a ruler
- scissors

GOAL

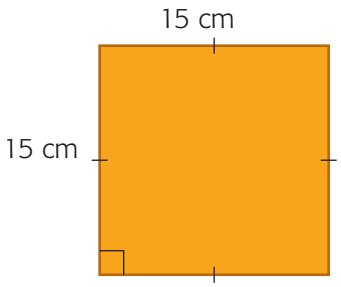
Develop and apply the formula for the area of a parallelogram.

LEARN ABOUT the Math

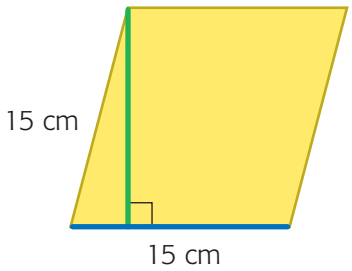
Matthew wrapped the lids of two gift boxes. One lid was a non-square rhombus, with each side measuring 15 cm. The other lid was a square, with each side measuring 15 cm. Matthew used different amounts of wrapping paper for each lid.

Which shape needed more wrapping paper?

A. Draw the square lid. How do you calculate its area?



B. Draw the rhombus-shaped lid. Draw its **height.** Measure its **base** and height.

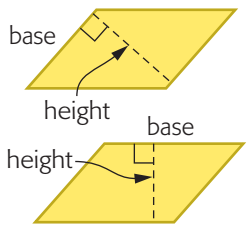


height

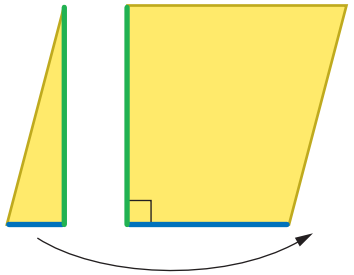
a line segment drawn to form a right angle with one side of a shape

base

the side of a shape that is measured to calculate the area or perimeter of the shape; any side of a shape can be the base of the shape



C. Cut the rhombus along its height, and move the triangular piece as shown. What do you notice about the new shape? How could you calculate its area?



D. What is the area of the rhombus?

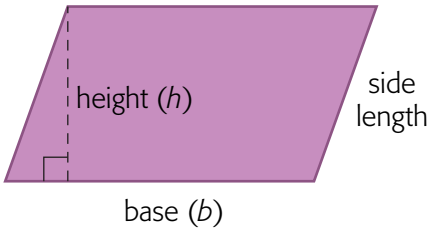
E. Which shape needed more wrapping paper? How do you know?

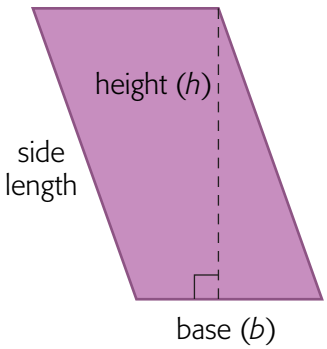
Communication | *Tip*

- The little square in the diagram above means that the two sides form a right angle (90°).
- Units of area usually have a small raised 2 written after them, as follows: 12 m^2. This indicates that the product of two dimensions, length and width or base and height, is involved in the measurement.

Reflecting

F. Can you cut any parallelogram and rearrange the pieces as in part C? Use some examples.





G. What is the **formula** for the area of a parallelogram with base b and height h? Explain.

formula

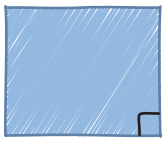
a rule represented by symbols, numbers, or letters, often in the form of an equation; for example, Area of a rectangle = length × width, or $A = l \times w$

WORK WITH the Math

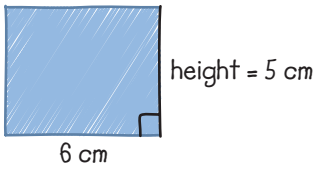
Example 1 | Drawing and measuring a parallelogram

Draw a parallelogram. Measure its base and height, and then determine its area.

Matthew's Solution



I drew a rectangle. A rectangle is one kind of parallelogram, and the height of a rectangle is easy to measure.



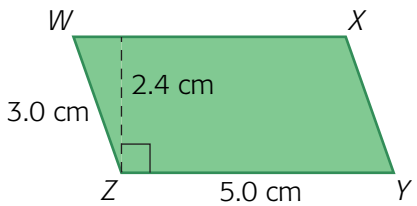
I measured the base and the height. The base is 6 cm, and the height is 5 cm.

$A = b \times h$
$= 6 \text{ cm} \times 5 \text{ cm}$
$= 30 \text{ cm}^2$
The area is 30 cm^2.

The area is the product of the base and the height. The area of my parallelogram is 30 cm^2.

Example 2 | Calculating the area of a parallelogram

Calculate the area of parallelogram *WXYZ*.

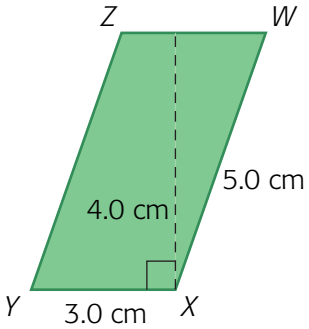


Julie's Solution

$A = b \times h$
$= 5.0 \text{ cm} \times 2.4 \text{ cm}$
$= 12.0 \text{ cm}^2$
The area of WXYZ is 12 cm^2.

The area of a parallelogram is base $\times$ height. I decided to use *ZY* as the base. This meant that I had to use 5.0 cm for the base and 2.4 cm for the height.

Max's Solution

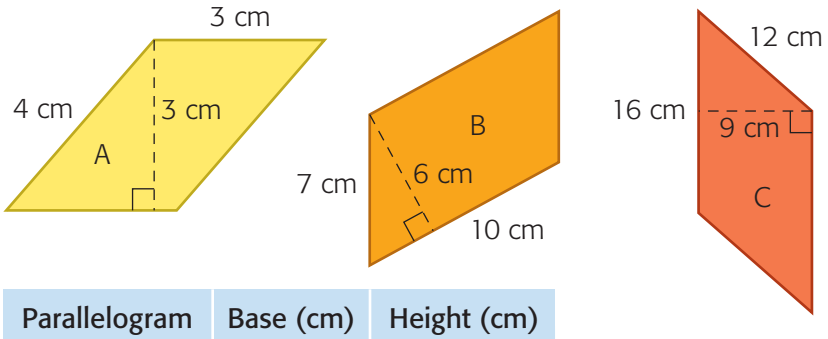


A = b x h
= 3.0 cm x 4.0 cm
= 12.0 cm^2
The area of WXYZ is 12 cm^2.

I decided to use *YX* as the base. This meant that I had to use 3.0 cm for the base and 4.0 cm for the height.

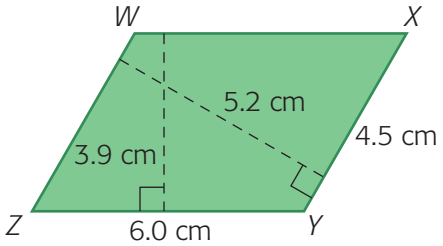
A Checking

1. Complete the table for the parallelograms shown.



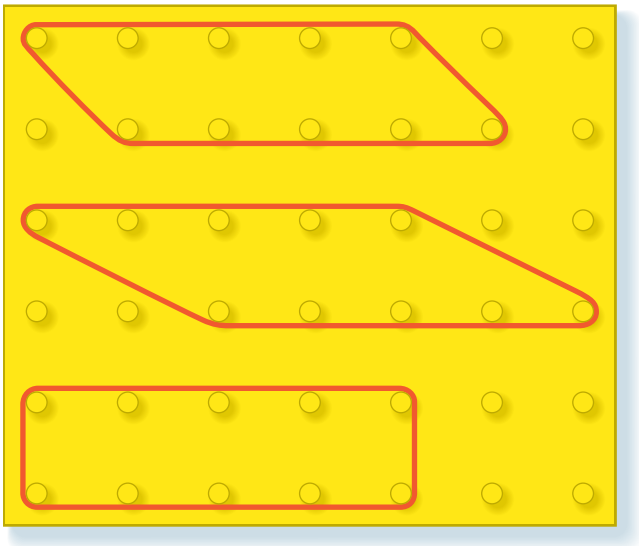
Parallelogram	Base (cm)	Height (cm)
A		
B		
C		

2. Calculate the area of parallelogram *WXYZ* using two different bases. Show your work.



B Practising

3. Why do all these geoboard shapes have the same area?



4. Create three parallelograms on a geoboard or dot paper.

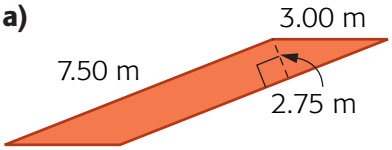
a) Show the height of each parallelogram. Record your results in a table.

b) Estimate the area of each parallelogram, in square units, by counting the squares.

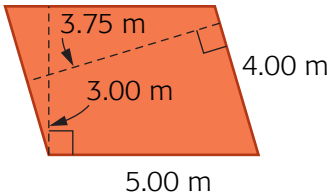
c) Calculate the area of each parallelogram, in square units, using a formula.

5. Calculate the area of each parallelogram to two decimal places.

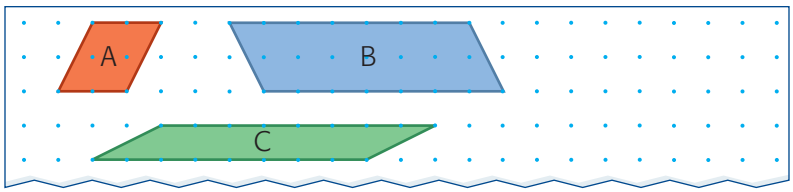
a)



b)



6. a) Draw each parallelogram on 1 cm square dot paper. Then label a base and the corresponding height.



b) Calculate the area of each parallelogram.

7. Complete the table.

	Base	Height	Area of parallelogram
a)	4 m	▢ m	28 m^2
b)	20 cm	11 cm	▢ cm^2
c)	▢ cm	9 cm	63 cm^2
d)	1.7 m	2.6 m	▢ m^2
e)	0.6 m	▢ m	4.2 m^2
f)	27.5 cm	32.6 cm	▢ cm^2

> ***Reading Strategy***
> Use what you already know about calculating the area of a rectangle to help you solve this problem.

8. Draw three different parallelograms, each with an area of 36 cm^2.

9. Draw a parallelogram, and label it A. Now draw two more parallelograms, as described below.
- **a)** a parallelogram with half the area of A
- **b)** a parallelogram with twice the area of A

10. Draw a parallelogram in which both the base and the height are sides. Explain your thinking.

11. Thrillville Amusement Park needs a parking lot. The parking spaces will be angled in a row. The owners of the park expect that most vehicles will be less than 5.0 m long and 2.8 m wide, with both doors open. The cost to pave is $1.25 for each square metre.
- **a)** Sketch the parking lot.
- **b)** Complete the following table. Show your calculations.

Number of parking spaces	Area (m^2)	Total cost ($)
1		
5		
10		
15		

12. Why do you not usually multiply the side lengths of a parallelogram to calculate its area?

5.2 The Area of a Triangle

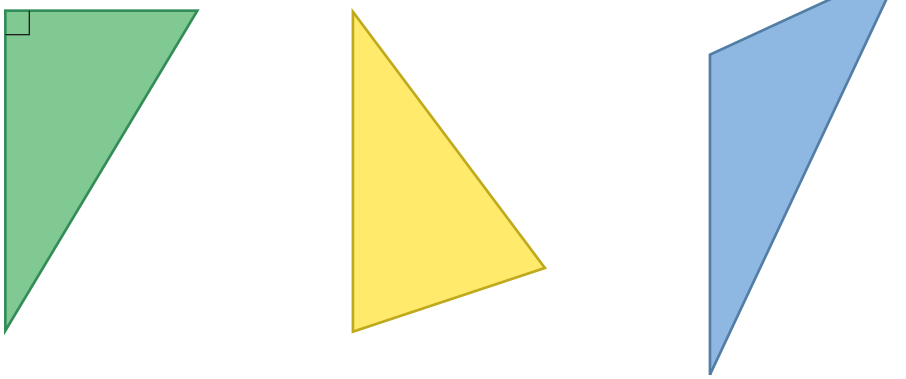
YOU WILL NEED
- graph paper

GOAL

Develop and apply the formula for the area of a triangle.

LEARN ABOUT the Math

Yan designed three class flags. The class decided to choose the biggest flag.

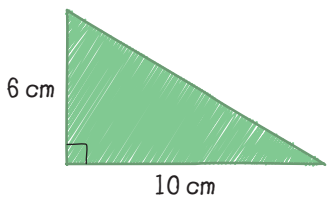


Which flag is the biggest?

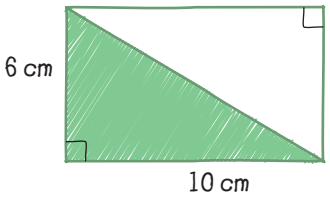
Example 1 | Calculating the area of a right triangle

Calculate the area of the green flag.

Yan's Solution



I measured the triangle. It's a right triangle.
It has a base of 10 cm and a height of 6 cm.



The triangle has a right angle, so I made a rectangle that includes it. The flag is half of the rectangle. The rectangle has the same base and height that the triangle does.

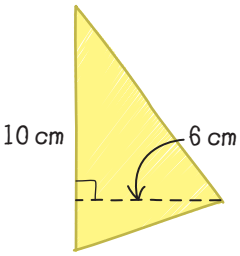
Area = (6 cm x 10 cm) ÷ 2
= 30 cm^2
The area of the green flag is 30 cm^2.

The area of the rectangle is 6 cm × 10 cm.
The area of the triangle is half of this.

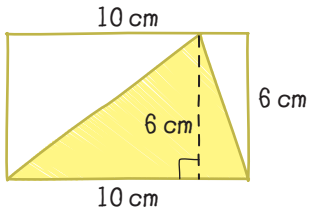
Example 2 | Calculating the area of an acute triangle

Calculate the area of the yellow flag.

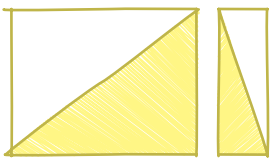
Liam's Solution



I measured the triangle. It's an acute triangle.
It has a base of 10 cm and a height of 6 cm.



I imagined the triangle as part of a rectangle with the same base and same height as the triangle.



I imagined the rectangle being split into two rectangles along the triangle's height.
I noticed that each part of the yellow triangle is half the area of each of the two new rectangles.

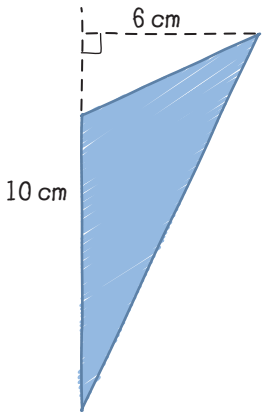
Area = (6 cm x 10 cm) ÷ 2
= 30 cm^2
The area of the yellow flag is 30 cm^2.

This meant that the area of the triangle must have been half the total area of the rectangle with the same base and same height.

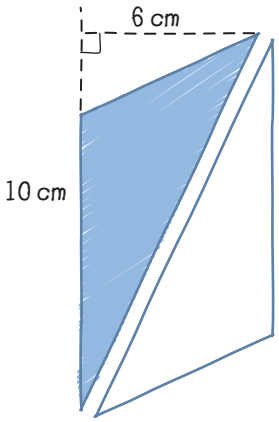
Example 3 | Calculating the area of an obtuse triangle

Calculate the area of the blue flag.

Nolan's Solution



I measured the triangle. It's an obtuse triangle.
It has a base of 10 cm.
The height of the triangle is 6 cm. I measured it outside the triangle.



It was hard to imagine the triangle as half of a rectangle. It was easier to imagine another identical triangle, arranged to form a parallelogram.
This parallelogram has the same base and same height as the triangle.

Area = (6 cm x 10 cm) ÷ 2
= 30 cm^2
The area of the blue flag is 30 cm^2.

All the flags have the same area, so the class can pick any one of them.

I thought about rearranging this parallelogram into a rectangle, but I knew that the area of a parallelogram is base × height. I multiplied 6 × 10, and then divided by 2. I didn't need to do the rearranging.

Reflecting

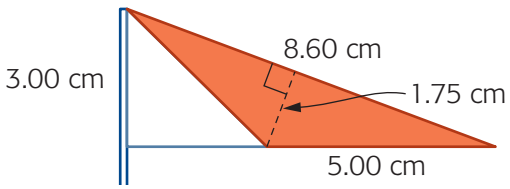
A. Can every triangle be shown as half of a parallelogram?

B. What is the formula for the area of a triangle?

WORK WITH the Math

Example 4 | Choosing dimensions to calculate area

Fiona designed this flag. Is her flag bigger or smaller than Yan's flags?



Solution

$A = (5.00 \text{ cm} \times 3.00 \text{ cm}) \div 2$
$= 7.50 \text{ cm}^2$

Use the 5.00 cm side as the base and 3.00 cm as the height.

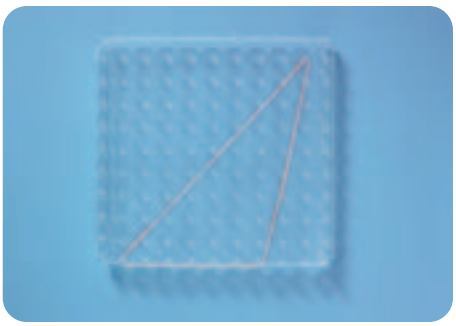
$A = (8.60 \text{ cm} \times 1.75 \text{ cm}) \div 2$
$= 7.53 \text{ cm}^2$

Or, use the 8.60 cm side as the base and 1.75 cm as the height.
The second answer is a little different from the first answer, probably because Fiona didn't measure as carefully as she should have. It's very close, though.

The area of Fiona's flag is about 7.5 cm^2.
This is a lot less than the area of Yan's flags.

Example 5 | Calculating area on a geoboard

Michael created a triangle on a geoboard. What is the area of the triangle?



Solution

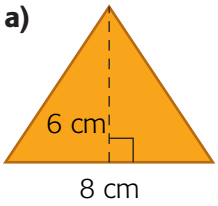
$A = (b \times h) \div 2$
$= (6 \times 9) \div 2$
$= 54 \div 2$
$= 27$ square units

The triangle is obtuse. Its base is from 1 to 7, or 6 units. Its height is from 1 to 10, or 9 units. Its area is the base multiplied by the height, divided by 2.

A Checking

1. Calculate the area of each triangle.

a)



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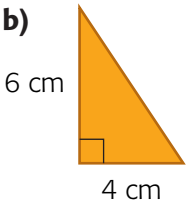
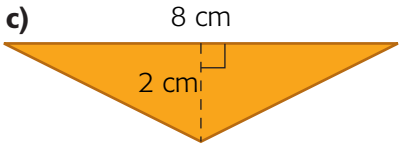


c)

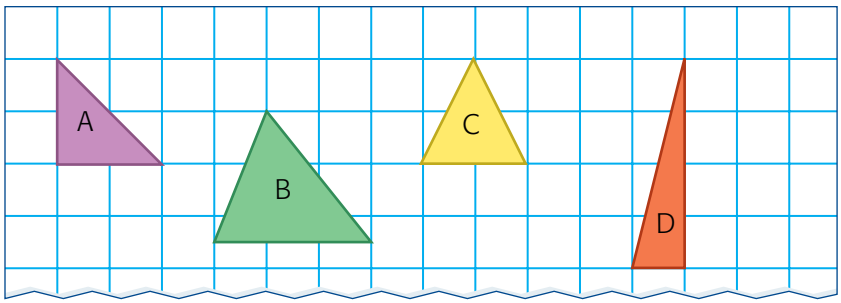


2. Complete the table.

	Base	Height	Area of triangle
a)	4 cm	9 cm	☐ cm^2
b)	12 cm	45 cm	☐ cm^2
c)	3.5 m	3.0 m	☐ m^2
d)	6.0 m	7.5 m	☐ m^2

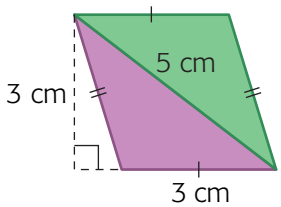
A Practising

3. Draw each triangle on centimetre grid paper and estimate its area. Based on your estimates, predict whether any of the triangles have the same area. Check by measuring and by calculating.



4. Calculate the area of each purple triangle.

a)



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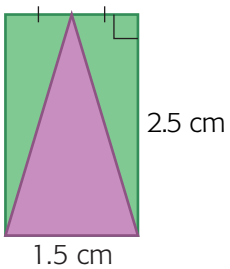
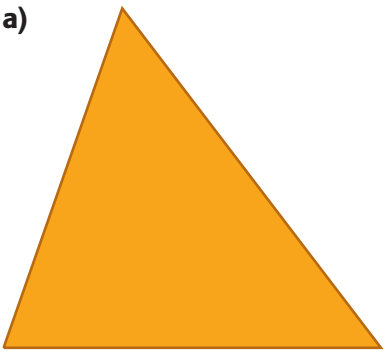


5. Complete the table.

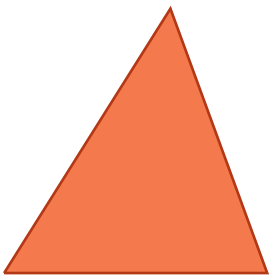
	Base	Height	Area of triangle
a)	6 cm	▢ cm	72 cm^2
b)	▢ m	6.0 m	10.2 m^2
c)	40 mm	9 cm	▢ cm^2
d)	250 mm	▢ cm	625 cm^2

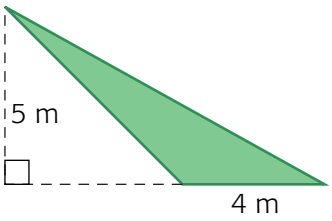
6. Measure a base and a height for each triangle. Then calculate the area of each triangle.

a)



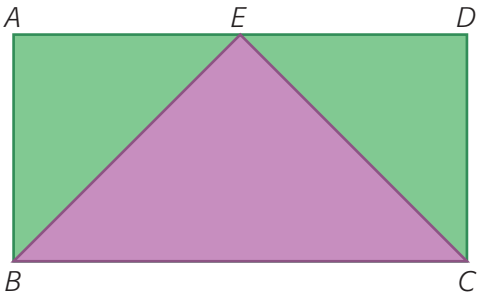
b)



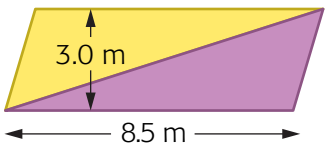


7. Calculate the area of the green triangle at the left.

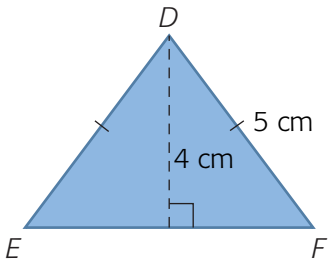
8. Calculate the area of each shape. Use a ruler to measure the sides.



a) $\triangle BEC$
b) $\triangle EDC$
c) $\triangle ABE$
d) rectangle $ABCD$



9. **a)** Calculate the area of the yellow fabric in the flag at the left.
b) The price of the yellow fabric is \$8.40/m^2. Calculate the cost of the yellow fabric for one flag.
c) Calculate the area of the yellow fabric needed for 10 flags.

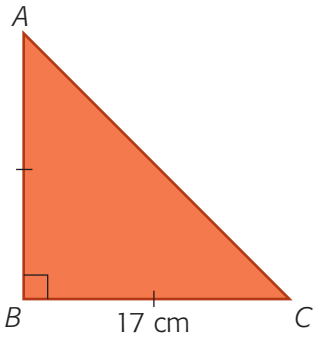


10. The perimeter of $\triangle DEF$ is 16 cm.

a) How long is EF?

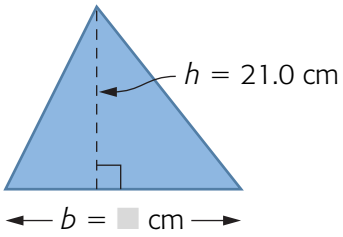
b) Calculate the area of $\triangle DEF$.

11. The perimeter of $\triangle ABC$ is 58 cm. Calculate the area of $\triangle ABC$.

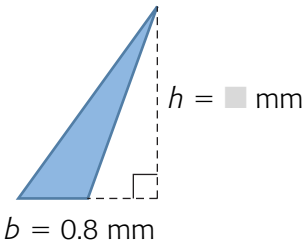


12. Determine each missing value.

a) $A = 283.5 \text{ cm}^2$



b) $A = 1.6 \text{ mm}^2$

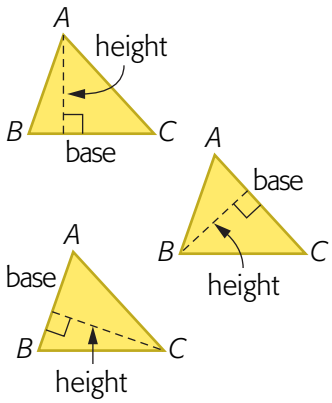


13. Design a kite by combining four triangles. Calculate the area of each triangle and the total area of the kite.

14. A triangle with an area of 8 cm^2 is twice as long and twice as high as another triangle. What is the area of the other triangle?

15. Can a triangle and a parallelogram have the same area? Explain.

16. Triangle A has two sides that measure 4 cm and one side that measures 5 cm. Triangle B has a base of 4 cm and a height of 5 cm. Which triangle has the greater area?



17. $\triangle ABC$ has three different pairs of bases and heights, as shown at the left.

a) Does every triangle have three different pairs of bases and heights? Explain.

b) When you use a formula to calculate the area of a triangle, does it matter which base-height pair you use? Explain.

5.3 Exploring Circumference and Diameter

YOU WILL NEED

- wheels
- a measuring tape or a metre stick

GOAL

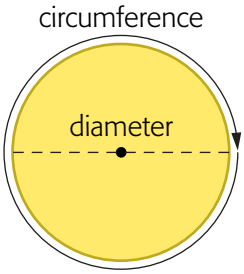
Investigate the relationship between the diameter and circumference of a circle.

circumference

the boundary of a circle; the length of this boundary

diameter

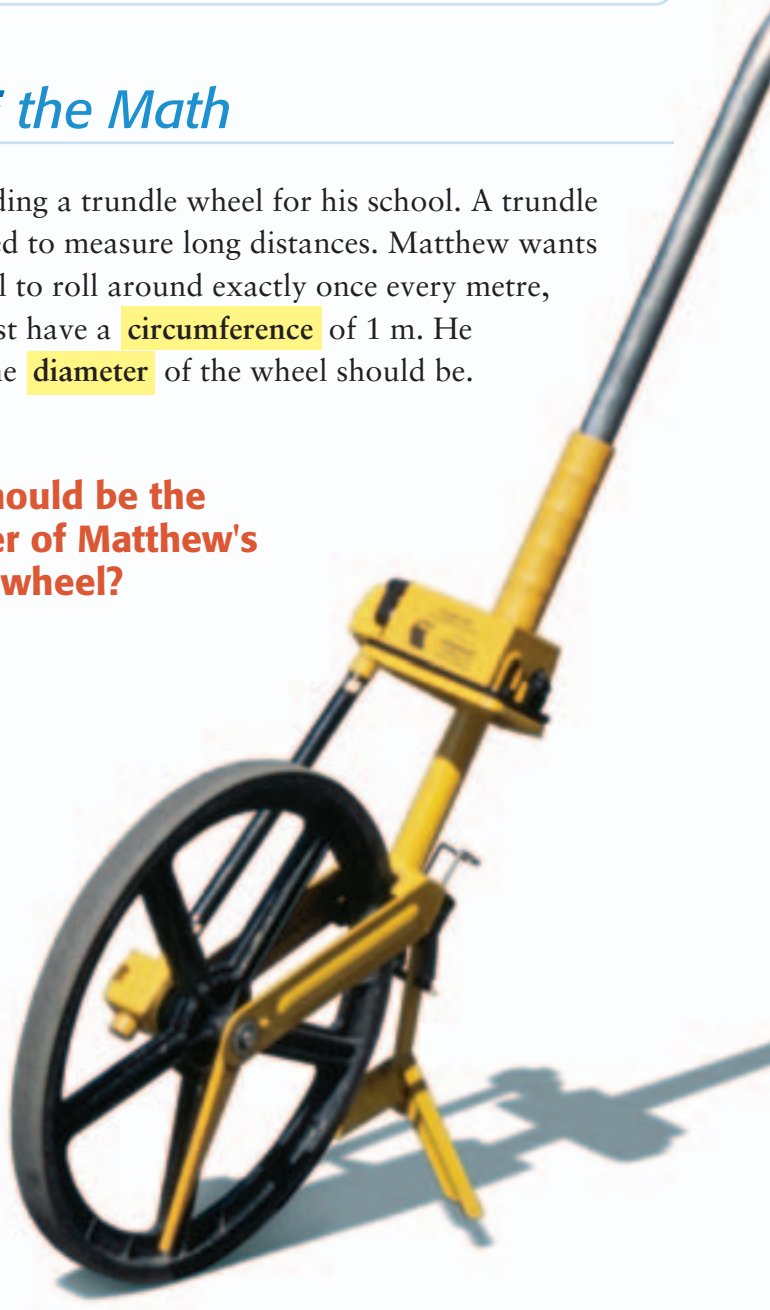
a line segment that joins two points on the circumference of a circle and passes through the centre; the length of this line segment; the diameter is the longest line segment that can be drawn inside a circle



EXPLORE the Math

Matthew is building a trundle wheel for his school. A trundle wheel can be used to measure long distances. Matthew wants his trundle wheel to roll around exactly once every metre, so the wheel must have a **circumference** of 1 m. He wonders what the **diameter** of the wheel should be.

What should be the diameter of Matthew's trundle wheel?



5.4 Calculating Circumference

YOU WILL NEED
- a calculator
- a compass

GOAL

Apply the formula for the circumference of a circle using π.

LEARN ABOUT the Math

Nolan is making dream catchers. He wants to know how to predict the length of the branches he needs for the circle frame.

How can you predict the circumference when you know the diameter?

Example 1 | Calculating circumference using diameter

Write a formula that uses the diameter of a circle to calculate its circumference.

Nolan's Solution

Diameter (cm)	Circumference (cm)
5.0	15.7
10.0	31.4
15.0	47.1
20.0	62.8

I modelled dream catchers with circles.
I traced circles that were different sizes and measured the diameters.
Then I measured the circumferences with string and a tape measure.
I recorded the measurements in a table.

15.7 : 5.0 = 3.14 : 1
31.4 : 10.0 = 3.14 : 1
47.1 : 15.0 = 3.14 : 1
62.8 : 20.0 = 3.14 : 1

I wrote the measurements as ratios.

Another way to write this is
Circumference : Diameter = π : 1
Each time,
$C = \pi \times d$

I calculated the ratio of the circumference to the diameter for each circle. I noticed they were all equal. My teacher told me that this ratio is called **π.**

π (pi)

the ratio of the circumference of a circle to its diameter; its value is about 3.14

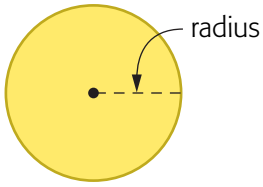
Communication | *Tip*

When you do calculations that involve measurements, it is important that the result not seem more precise than the measurements, so you should round the results. In this book, round your final result to the same number of decimal places that are in the least precise measurement.

For example, $1.60 \text{ m} \times 1.24 \text{ m} = 1.984$
round to two decimal places $= 1.98 \text{ m}^2$

radius

half of a diameter; the distance from the centre of a circle to a point on the circumference

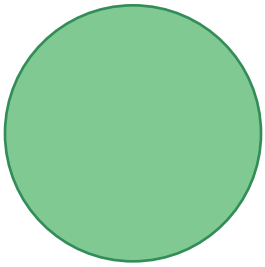


Reflecting

A. How can you estimate the circumference of a circle when you know its **radius** instead of its diameter?

B. How can you estimate the diameter of a circle when you know its circumference?

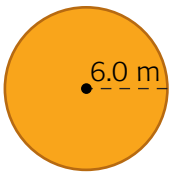
C. Suppose that you had to determine the circumference of the following circle. Would you use a formula with the diameter, or would you measure the circumference? Explain your choice.



WORK WITH the Math

Example 2 | Determining circumference using a calculator

Determine the circumference of this circle.



Solution

$C = \pi d$

Use the formula for circumference.
The radius of the circle is 6.0 m, so its diameter is 12.0 m.

1 2 . 0 × π =

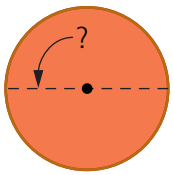
Multiply the diameter by π.
Use the π key on your calculator. If you don't have a π key, estimate with 3.14.

The circumference is about 37.7 m.

Answer to one decimal place because the radius is given to one decimal place.

Example 3 | Determining diameter

This circle has a circumference of 10.0 cm.
Determine its diameter.



Solution

$C = \pi \times d$ $10.0 \text{ cm} = \pi \times d$	Use the formula. Substitute in the measurement you know.
$d = 10.0 \text{ cm} \div \pi$ $= 3.184 \ldots$	The circumference is equal to $\pi \times d$, so the diameter must be equal to the circumference divided by π. Use a calculator. Use the $\boxed{\pi}$ key or 3.14.
The circumference is about 3.2 cm.	Answer to one decimal place because the circumference is given to one decimal place.

A Checking

1. Determine the circumference of a circle with each diameter.

a) 5 cm **b)** 4.7 cm

2. Determine the diameter and the circumference of a circle with each radius.

a) 10 cm **b)** 8.2 m

B Practising

3. Determine the circumference of a circle with each diameter.

a) 4.5 cm **c)** 6.4 cm **e)** 7 mm
b) 1.7 cm **d)** 36.0 m **f)** 4.0 cm

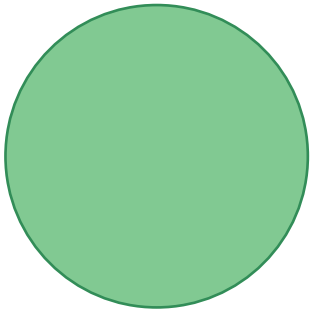
4. Determine the circumference of a circle with each radius.

a) 7 mm **c)** 6.3 cm **e)** 23.1 m
b) 19.5 cm **d)** 9.0 cm **f)** 0.05 m

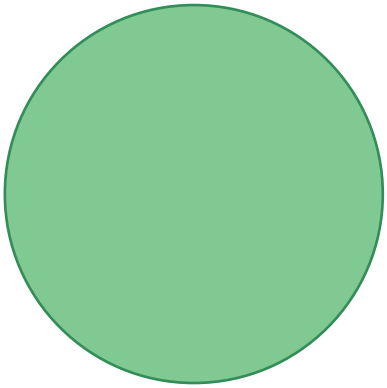
5. The diameter of each wheel on Xavier's bicycle is 80 cm. Determine the circumference of each wheel.

6. Measure the diameter of each circle. Then determine the circumference.

a)

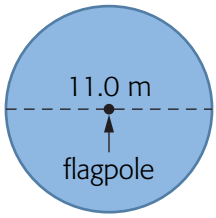


b)



7. What is the diameter of a circle with a circumference of 30.0 cm?

8. At summer camp, Maria uses chalk to draw a meeting circle around a flagpole. The distance from one side of the circle to the other through the flagpole in the centre is 11.0 m. What is the circumference of the chalk circle?



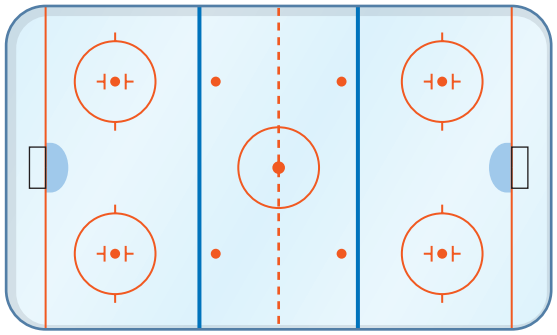
9. Complete the table.

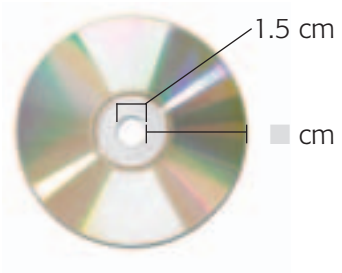
Item	*r*	*d*	*C*
clock	9.0 cm		
watch		36 mm	
round tea bag	1.9 cm		
sewer cover		62 cm	
circle protractor	5.9 cm		
electric fan		201 mm	

10. At a zoo, the giraffes are fenced inside a circular field with a radius of 700 m. How long is the fence?

11. The bicycle at the left is called a penny-farthing. In one model, the diameter of the front wheel is 120.0 cm. In another model, the diameter is 150.0 cm. What is the difference in the circumferences of these front wheels?

12. For hockey practice, Rosa has to skate around a faceoff circle five times. The faceoff circle has a diameter of 9.0 m. About how far does Rosa have to skate?

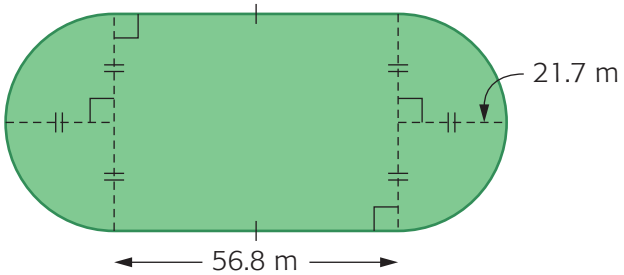


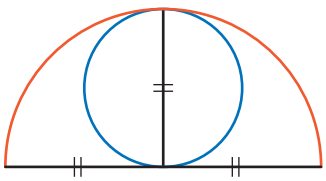


13. The circumference of the CD at the left is 37.7 cm. The diameter of the hole in the centre is 1.5 cm. What is the distance from the outside edge of the CD to the inside edge?

14. One of the largest trees in the world is the giant sequoia. It is more than 90 m tall. The diameter of one giant sequoia is 9.2 m. What is the circumference of its trunk?

15. This racetrack consists of a rectangle and two half circles. What is the length of one lap?

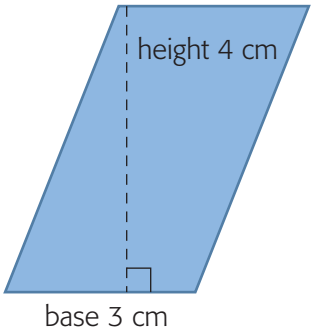


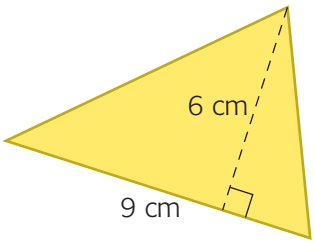


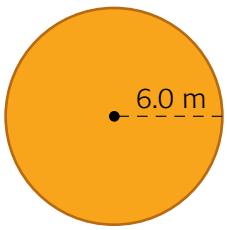
16. The radius of the blue circle at the left is 8 m. Determine the red half-circumference.

17. Brian says that two circles with the same radius can have different circumferences. Do you agree or disagree? Explain.

Mid-Chapter Review







Frequently Asked Questions

Q: How can you calculate the area of a parallelogram?

A: You can use the formula $A = b \times h$. The height (h) is the perpendicular distance from the base (b) to the opposite side. For example, the area of this parallelogram is $3 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2$.

Q: How can you calculate the area of a triangle?

A: You can use the formula $A = (b \times h) \div 2$. For example, the area of this triangle is $(9 \text{ cm} \times 6 \text{ cm}) \div 2 = 27 \text{ cm}^2$.

Q: How can you determine the circumference of a circle using the diameter of the circle?

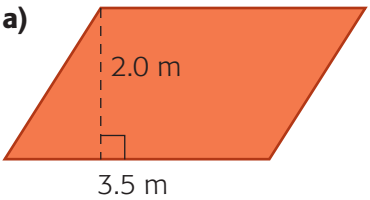
A: You can use the formula $C = \pi \times d$ or $C = 2 \times \pi \times r$, with 3.14 as an approximate value for π. π (pi) is the ratio of the circumference of a circle to its diameter. The value of π is 3.14, expressed to two decimal places. For example, the circumference of the circle at the left is about $2 \times 3.14 \times 6.0 \text{ m} = 37.7 \text{ m}$.

Practice

Lesson 5.1

1. Calculate the area of each parallelogram. Measure, if necessary.

a)



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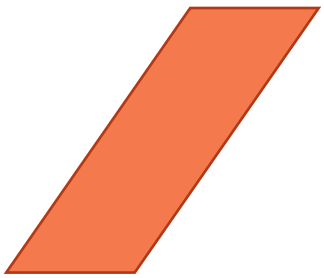
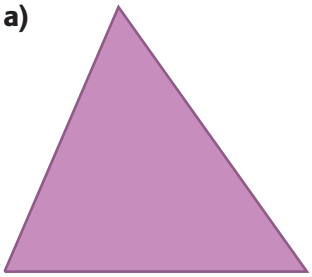


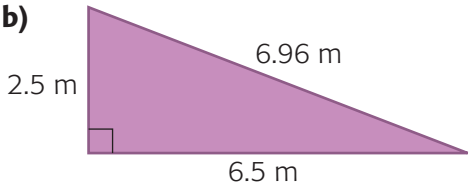
Lesson 5.2

2. Calculate the area of each triangle. Measure, if necessary.

a)



b)



Lesson 5.4

3. Determine the circumference of a circle with each diameter.

a) 26 cm **b)** 10.8 m **c)** 17.2 cm **d)** 3 km

4. What is the circumference of the circle in each sign?

a)

c)

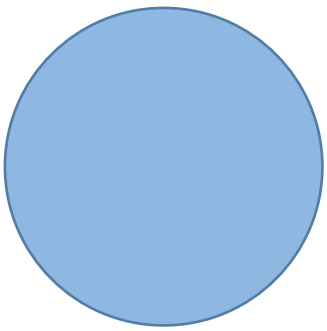
e)

b)

d)

f)

5. Measure the diameter of this circle. Then determine its circumference.



5.5 Estimating the Area of a Circle

YOU WILL NEED

- a compass
- a ruler
- scissors
- centimetre grid paper
- glue or tape
- a CD

GOAL

Estimate the area of a circle.

LEARN ABOUT the Math

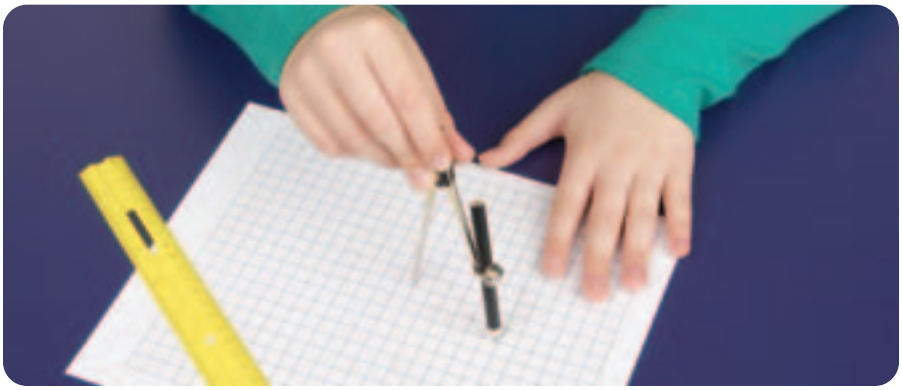
Julie's school has made a CD of their concert. The school plans to sell the CDs as a fundraiser. Each student in Julie's class needs to design a CD label, which will be printed on sheets of label paper and then cut out. Julie wants to know the area of one CD label so that she can detemine how many label sheets she needs to buy for the class.

How can you estimate the area of a CD label?

A. Measure the diameter of a CD. Then determine its radius.

B. Make a paper model of the CD. Adjust a compass so that the distance between the compass point and the pencil tip is equal to the radius. Use a ruler to adjust the compass.

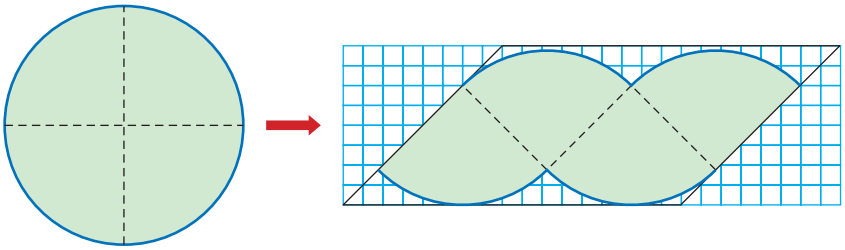
C. Put the compass point where you want the centre of the circle to be, and draw the circle.



D. Cut out the circle. You now have a model of the CD.

E. Fold your model into quarters, and then cut out the four equal sections.

F. Form the sections into a shape that almost fills a parallelogram, as shown. Draw the parallelogram.



G. Record your data in the following table.

Number of sections	Length of base of parallelogram	Height of parallelogram	Area of parallelogram
4			
8			
16			

H. Make two more models of the CD. Fold one into 8 equal sections and the other into 16 equal sections. Repeat parts F and G for each model.

I. Estimate the area of a CD label using the three values for the area of a parallelogram.

Reflecting

J. In part B, why did you have to adjust your compass to the size of the radius to create the circle?

K. Why is the area of a parallelogram with more sections a better estimate for the area of the circle?

L. What information about the circle do you need in order to calculate the area of the parallelogram that best estimates the area of the circle?

Reading Strategy

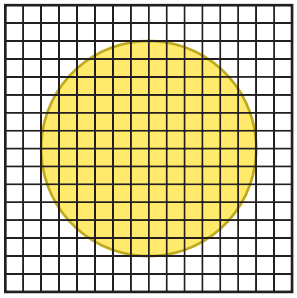
Share your answer with a partner. Can your partner add any new ideas or information?

WORK WITH the Math

Example 1 | Estimating the area of a circle

Estimate the area of the CD label by counting grid squares.

Julie's Solution



I traced around a CD on centimetre grid paper. I counted the number of whole squares inside the circle. There are 96. Then I mentally combined parts of squares to make whole squares. There are about another 12 squares.

The area is about $96 + 12 = 108\ cm^2$.

Example 2 | Estimating the area of a circle with squares

Estimate the area of the CD label by enclosing it with squares.

Max's Solution

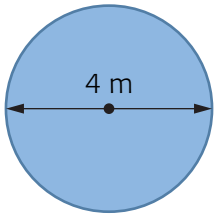
I traced a model on paper using the CD.

I drew a large square outside the circle and a small square inside the circle.

The area of the large square is $12\ cm \times 12\ cm = 144\ cm^2$.
The area of the small square is $8\ cm \times 8\ cm = 64\ cm^2$.
The area of the CD must be between these areas, or about $104\ cm^2$.

A Checking

1. Estimate the area of the face of each coin using the method in one of the examples.



B Practising

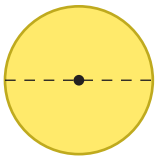
2. The distance across a park fountain is 4 m, as shown at the left. Estimate the area of the cover used during the winter.

3. Katie is building a wall clock in the design and technology club. The minute hand is 15 cm long, and the clock face is flat. About how much glass will she need to cover the clock face?

4. A new circular wading pool at an amusement park is tiled with a design on the bottom. The wading pool is 5 m across. Estimate the area that the tile will need to cover.

5. Estimate the area of each circle.

a)



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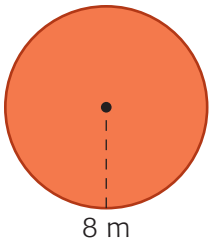
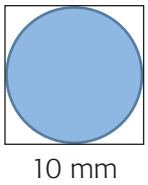
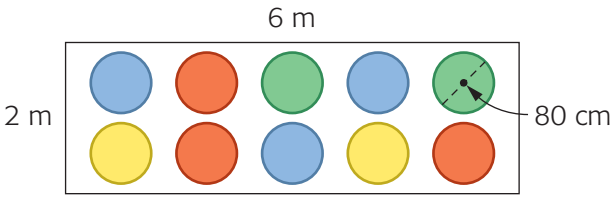


c)



6. About what fraction of the game mat is covered in circles?



7. One way to estimate the area of a circle is to cut the circle into equal sections and rearrange the sections to form a parallellogram. Another way is to count squares on grid paper. Which way do you think gives the better estimate? Why?

5.6 Calculating the Area of a Circle

YOU WILL NEED

- a calculator
- a compass
- scissors

GOAL

Develop and apply the formula for the area of a circle.

LEARN ABOUT the Math

Yan is setting up sprinklers to water a field. Each sprinkler sprays water in a circle with a diameter of 8.0 m.

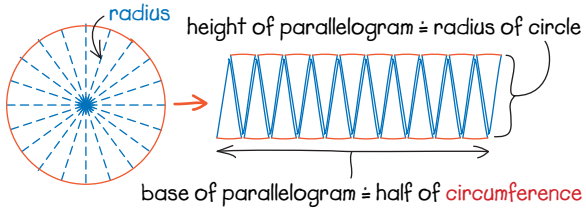
What area of the field does each sprinkler water?

Example 1 | Estimating the area of a circle using a model

I decided a rough estimate is good enough.

Yan's Solution

My circle had a diameter of 8.0 cm. Each 1 cm represents 1 m. I cut my circle into 20 sections and arranged the sections into a parallelogram.



I drew a circle to represent the spray.

The height of the parallelogram is about 4.0 cm. This is the same as the radius of the circle.
The base of the parallelogram is about 12.0 cm. This is the same as half the circumference of the circle.

$A = b \times h$
$= 12.0\ m \times 4.0\ m$
$= 48.0\ m^2$

The area of the parallelogram is about $48.0\ m^2$.
Therefore, the area of the circle is also about $48.0\ m^2$.

I calculated the area of the parallelogram.

Example 2 | Estimating area using a formula

I wanted a more exact value for the area than a rough estimate, so I developed a formula.

Liam's Solution

The formula for the area of a parallelogram is $A = b \times h$.

Since the base of the parallelogram is half the circumference of the circle, and the height is the radius, I can write the formula as $A = (C \div 2) \times r$.

$C \div 2$ is half the circumference. Half of $\pi \times$ diameter is the same as $\pi \times$ radius. So, I can rewrite the formula for the area of a parallelogram as $A = \pi \times r \times r$.

$A = \pi \times r \times r$
$= 3.14 \times 4.0\ m \times 4.0\ m$
$= 3.14 \times 16.0\ m^2$
$= 50.2\ m^2$
The area watered by each sprinkler is about $50.2\ m^2$.

I used Yan's parallelogram to develop a formula.

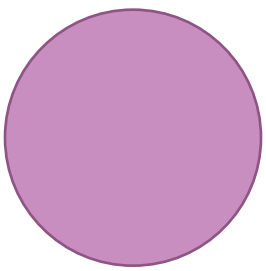
The height of the parallelogram was about the same as the radius of the circle.
The base of the parallelogram was about half the circumference of the circle.
I didn't want to calculate the circumference if I didn't have to, so I used the idea that the circumference is really $\pi \times$ diameter.
The radius is half the diameter, so the radius is easier to use in my formula.

I used my formula to calculate the area of the circle.
The diameter of the circle is 8.0 m, so the radius is 4.0 m.
I answered to the same number of decimal places as given in the problem.

Communication | *Tip*

You might see the formula for the area of a circle written as $A = \pi r^2$. The raised 2 after the r means that r is multiplied by itself ($r^2 = r \times r$).

Reflecting



A. What do you need to know about a circle to calculate its area?

B. Suppose that you had to determine the area of the circle at the left. Would you use a formula with the radius, or only measurements? Explain your choice.

WORK WITH the Math

Example 3 | Calculating area using the π key

Calculate the area of a circle with a radius of 9.0 cm.

Solution

Use the formula for the area of a circle.
$A = \pi \times r \times r$
$= \pi \times 9.0 \text{ cm} \times 9.0 \text{ cm}$
$= \pi \times 81.0 \text{ cm}^2$
$=$ 

Multiply the radius by itself.
Multiply this product by π. Use the π key on your calculator.

The area of the circle is 254.5 cm^2.

Answer to one decimal place.

Example 4 | Using an estimate for π

A circular garden stone is needed for the school environment project. The garden stone has a diameter of 5.0 m. What is the area of the garden stone?

Solution

Use the formula for the area of a circle.
$A = \pi \times r \times r$
$= 3.14 \times 2.5 \text{ m} \times 2.5 \text{ m}$
$= 3.14 \times 6.25 \text{ m}^2$
$= 19.625 \text{ m}^2$

The radius of the stone is half the diameter, or 2.5 m.
Multiply the radius by itself.
Use the approximate value for π, which is 3.14.

The area of the garden stone is about 19.6 m^2.

Answer to one decimal place.

A Checking

1. Estimate the area of each object using the formula $A = \pi \times r \times r$.

a)

radius = 10.5 cm

c)

radius = 13 cm

b)

diameter = 14 cm

d)

diameter = 2.8 cm

B Practising

2. Calculate the area of a circle with each measurement.

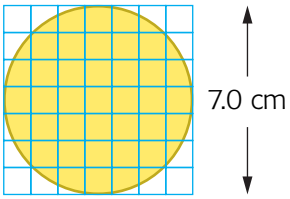
a) diameter = 7.3 cm

b) radius = 2 cm

c) radius = 2.7 cm

d) diameter = 1.7 cm

3. Estimate the area of the circle at the left, with and without using a formula. Explain what you did.

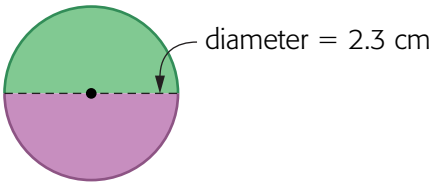


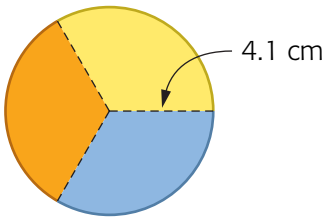
4. a) Describe a situation where you would need to know the area of a circle.

b) Describe a situation where you would need to know the circumference of a circle.

5. a) Determine the area of this circle using a formula.

b) Determine the area of each section.





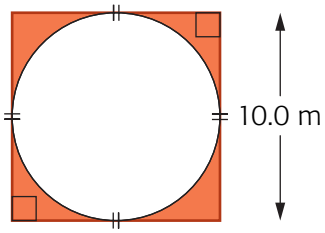
6. a) Determine the area of the circle at the left.

b) The three sections are equal. Determine the area of each section.

7. The radius of a circular pizza is 22.0 cm.

a) Determine the area of the pizza.

b) The pizza is cut into four equal pieces. Determine the area of each piece.

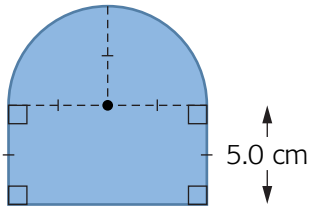


8. **a)** Determine the area of the square at the left.

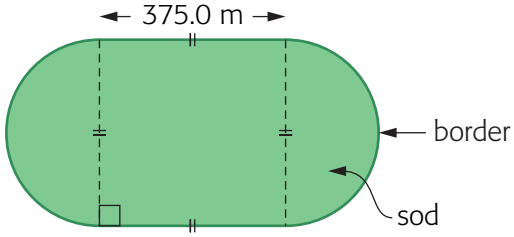
b) Determine the area of the white circle.

c) Determine the total area of the four red sections.

9. Determine the total area of this figure.



10. Roberto has designed the following park for a new housing development. The park will be a square with a half circle at each end. It will be covered with sod and have a border made of paving stones.



a) Determine the area of the sod needed to cover the park.

b) Sod costs $1.25 for each square metre. Determine the cost of the sod needed to cover the park.

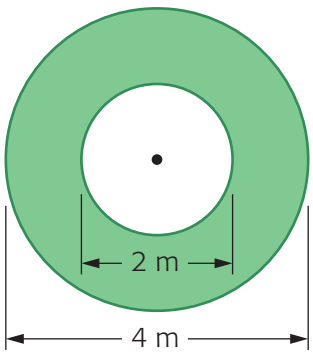
c) Determine the length of the border.

d) Paving stones cost $2.75 for each square metre. Determine the cost of the border.

e) Determine the total cost of the sod and the border.

11. Suppose that the radius of a circle doubles. How does the area change?

12. How would you calculate the area of the green ring at the left?



13. Explain the steps you would use to calculate the area of a circle with a circumference of 10.0 cm.

CURIOUS MATH

Pick's Theorem

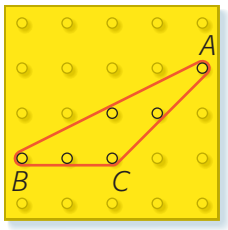
You can calculate the areas of some shapes on a geoboard by counting squares or using a formula. You can calculate the area of *any* enclosed shape on a geoboard, however, using *Pick's theorem*.

Using Pick's Theorem

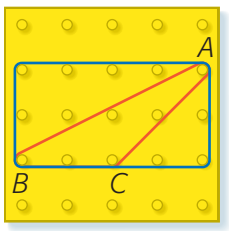
Step 1: Count the number of pegs (p) that the elastic band touches.
Step 2: Divide by 2.
Step 3: Add the number of pegs (i) inside the shape.
Step 4: Subtract 1.

For example, $\triangle ABC$ has 6 perimeter pegs and 0 interior pegs.

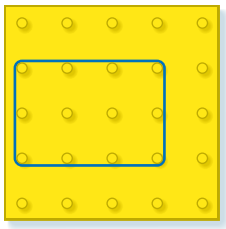
Area of $\triangle ABC = p \div 2 + i - 1$
$= 6 \div 2 + 0 - 1$
$= 2$ square units

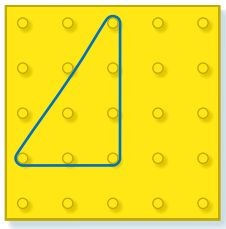


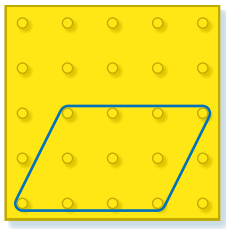
1. Check the area of $\triangle ABC$.
- **a)** Calculate the area of the rectangle around $\triangle ABC$.
- **b)** Calculate the area of each blue triangle.
- **c)** Subtract the total area of the blue triangles from the area of the rectangle. Does your answer agree with the answer using Pick's theorem?



2. Calculate the area of each shape using Pick's theorem. Check by counting squares or using a formula.

a)


b)


c)


5.7 Solve Problems Using Diagrams

YOU WILL NEED

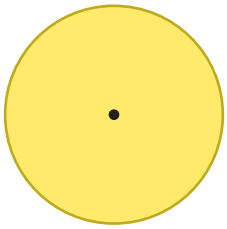
- pattern blocks
- a ruler
- a calculator
- a compass
- a protractor

GOAL

Use diagrams to solve problems about the number of degrees in a circle.

LEARN ABOUT the Math

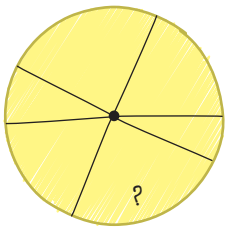
Julie is trying to divide a circle with a marked centre into six equal sections so that she can model the fraction $\frac{5}{6}$.



How can Julie divide the circle using her protractor?

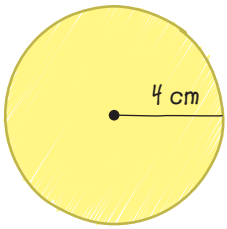
1 Understand the Problem

Julie draws a sketch. She knows that doing this will help her figure out how many degrees she should make the angles at the centre of the circle.



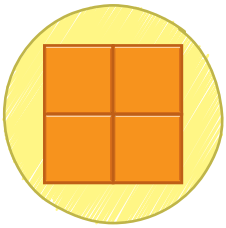
2 Make a Plan

Julie decides to use pattern blocks to help. She draws a circle with a radius of 4 cm on a piece of paper. She chooses square and triangular pattern blocks to investigate the number of degrees in a circle.



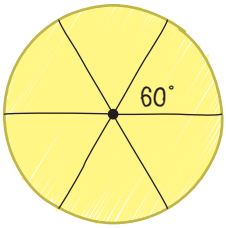
3 Carry Out the Plan

First Julie tries the squares. She puts a vertex of each square on the centre dot of the circle. Only four squares fit.



Julie knows that the angle at each vertex of the square is 90°. She realizes that the total of the angles in the centre must be $4 \times 90° = 360°$, but there are only four sections. She needs to divide the circle into six sections.
She decides to try the green triangles. Six green triangles fit.

The triangles are equilateral, so she knows that the angle at each vertex in the centre is 60°. The total of the angles in the centre must be $6 \times 60° = 360°$.



Julie says, "Since the total measure of the angles in the centre of the circle is 360°, each of my sections has to be 60°."

Reflecting

A. How did Julie's diagrams model the angle relationships of the pattern blocks and the circles?

B. How did Julie's original sketch support her method of solving the problem?

WORK WITH the Math

Example | Using diagrams to represent possible solutions

Matthew wants to use 16 square paint-sample cards to create a design for an art project. He wants to use a gold cord to frame the design, but he does not have very much cord. How can Matthew arrange the paint cards so there is the least amount of trim around the design?

Matthew's Solution

1 Understand the Problem

Matthew has to determine the perimeter of some possible designs to figure out if he has enough cord to go around the designs.

2 Make a Plan

Matthew decides to arrange the paint-sample cards in different ways to figure out which design has the least perimeter.

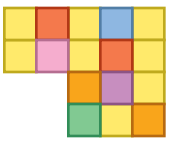
3 Carry Out the Plan

Matthew tries some possible designs.

$P = 16 + 16 + 1 + 1$
$= 34$

$P = 4 + 4 + 4 + 4$
$= 16$

$P = 8 + 8 + 2 + 2$
$= 16 + 4$
$= 20$



$P = 5 + 4 + 3 + 2 + 2 + 2$
$= 9 + 3 + 6$
$= 12 + 6$
$= 18$

Of these four designs, the 4×4 tile design has the least perimeter because more sides of the cards are inside the design. Matthew decides to use a square shape for his design.

4 Look Back

Matthew knows he did not try every possible design because he noticed that the perimeter increased when the design became longer. He thinks that the square design will always have the least perimeter because more card sides are inside that design.

A Checking

1. Arlene is making a stained-glass window using a rectangle and a half circle, as shown. How much glass will Arlene need?

B Practising

2. A rectangular wading pool at a park measures 6 m by 8 m. Around the pool, there is a tiled border that measures 10 m by 12 m. What is the area of the tiled border around the pool?

3. Julie is estimating the amount of paint she needs for the walls of her 3.4 m by 2.6 m bedroom. Her bedroom is 2.7 m high. One litre of paint covers about 10 m^2. About how much paint does Julie need?

4. Parallelogram A has a base of 10 cm and a height of 50 cm. Parallelogram B is 3 cm higher and has a base that is 5 cm longer. How much greater is the area of parallelogram B?

5. How many different ways can a 360-player marching band be arranged in a rectangle?

6. A tan pattern block has a 30° angle. How many of these blocks can be put together to meet at a point?

7. Sarah cares for 24 cats at the local animal refuge. Six of these cats have short tails, 12 are black, and 15 have long hair. All the cats in the refuge have at least one of these features. One cat is black and has a short tail and long hair. Two cats are black and have short tails, but do not have long hair. Two cats have short tails and long hair, but are not black. How many cats are black and have long hair, but do not have short tails?

MATH GAME

Rolling Circles

In this game, you will estimate the area of a circle, given its radius. Then you will try to form a two-digit number that is close to this area.

Number of players: 2 or more

YOU WILL NEED

- a 10-sided die numbered 0 to 9
- a pair of dice numbered 1 to 6
- a calculator

How to Play

1. Roll the 10-sided die. The number you roll represents the radius of a circle. Estimate the area of the circle.
2. Roll the pair of dice. Use the numbers you roll to form any two-digit number.
3. If the difference between the area of the circle and the two-digit number is 10 or less, score 4 points.
   If the difference is between 10 and 20, score 2 points.
   If the difference is 20 or greater, score 1 point.
4. Take turns. The first player to score 10 points wins.

Nolan's Turn

I rolled a 2 with the 10-sided die.

The area of the circle is $\pi \times 2 \times 2$, which is about $3 \times 2 \times 2$, or 12 square units.

Then I rolled a 3 and a 1 with the regular dice.

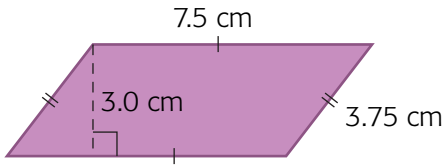
I can make 13 or 31. I'll make 13.

$13 - 12 = 1$
$1 < 10$, so I score 4 points.

Chapter Self-Test

1. Calculate the area of each parallelogram.

a)



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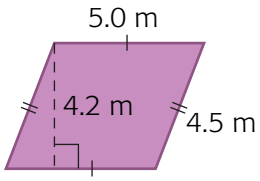
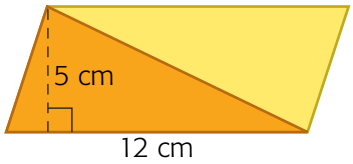


2. Calculate the area of the orange triangle in each parallelogram.

a)



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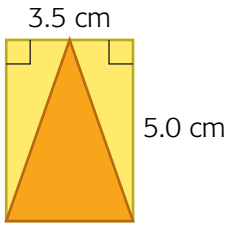

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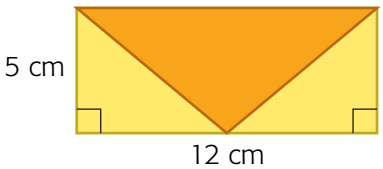
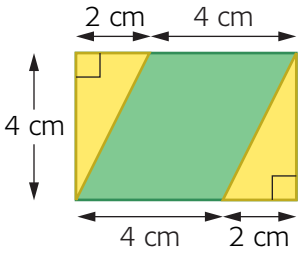


3. Calculate the area of the green shape in each rectangle.

a)



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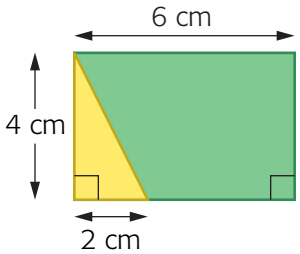


4. State whether each measurement is an area or a circumference.

a) the amount of sod needed to cover the circular green on a golf course

b) the amount of material needed to make a pool cover

c) the length of stone used for the border of a round garden

5. Determine the circumference of a circle with each measurement.

a) radius = 2.5 km

b) radius = 26 cm

c) diameter = 3.0 cm

d) diameter = 21 cm

6. Calculate the area of a circle with each measurement.

a) radius = 2 cm

b) diameter = 11 cm

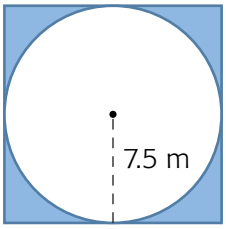
c) diameter = 5.7 cm

d) radius = 6.2 cm

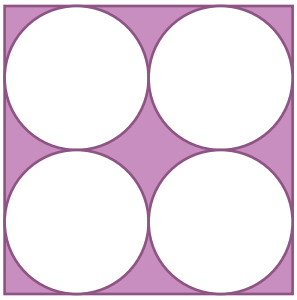
7. a) Determine the area of the square.

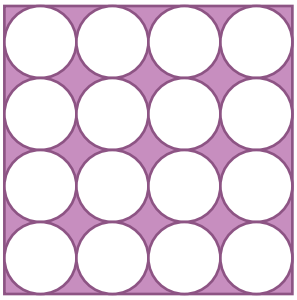
b) Determine the area of the white circle.

c) Determine the area of the blue sections.



8. Suppose that you cut circles in two pieces of cloth, as shown below. Which piece would have more cloth left?



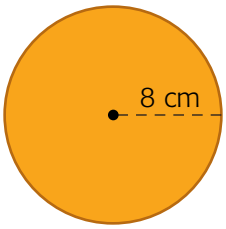


What Do You Think Now?

Revisit What Do You Think? on page 193. How have your answers and explanations changed?

Chapter Review

Frequently Asked Questions



Q: How do you determine the area of a circle?

A: You can use the formula $A = \pi \times r \times r$. For example, consider the circle at the left:

$A = 3.14 \times 8 \text{ cm} \times 8 \text{ cm}$
$= 201 \text{ cm}^2$

The area is about 201 cm^2.

Q. How can you draw a circle when you know its radius or diameter?

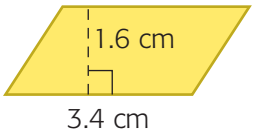
A: Adjust a compass so that the distance between the compass point and the pencil tip is equal to the radius. Put the compass point where you want the centre of the circle to be, and draw the circle.

Practice

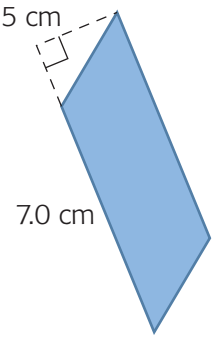
Lesson 5.1

1. Calculate the area of each parallelogram.

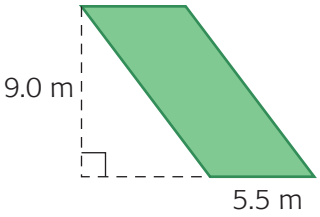
a)



b)



c)

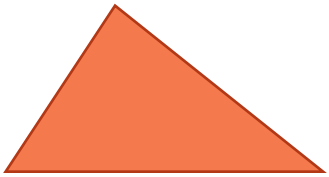


2. Determine the area of this parallelogram.

Lesson 5.2

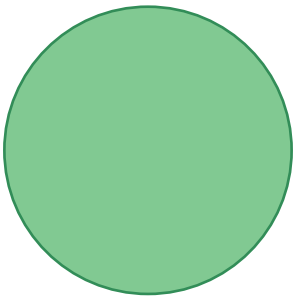
3. **a)** Draw three different triangles, each with a height of 5 cm.
b) Calculate the area of each triangle. Explain what you did.

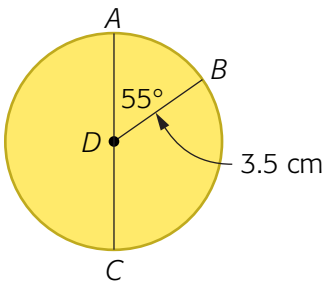
4. Determine the area of this triangle.



Lesson 5.4

5. Determine the circumference of this circle.





6. Determine the circumference of the circle at the left.

Lesson 5.5

7. Draw the circle in question 6 using a compass.

Lesson 5.6

8. Determine the area of the circle in question 6.

Lesson 5.7

9. **a)** Determine the measure of $\angle ADC$ in the circle in question 6.
b) Determine the measure of $\angle BDC$ in the circle in question 6.

Chapter Task

Task | *Checklist*

- ✔ Did you include all the required diagrams and sketches?
- ✔ Did you include all the required shapes in your design?
- ✔ Did you prepare a table that accurately lists the measurements required for the design?
- ✔ Did you use correct math language?

Portable Player Design

Portable players have buttons to select, play, rewind, fast forward, and stop music files. Suppose that you were asked to design the face of a portable player to win a prize.

How will you use parallelograms, triangles, and circles in your portable player design?

A. Design a portable player that will be appealing to customers.

B. Your design entry must include

- a diagram of the face of the player—the face must include parallelograms, triangles, and circles, and it must have an area of 60 cm^2 or less
- a circle divided into sections, with the measure of the vertex of each section
- a table that includes the percent of the total area of the face used by each different-shaped button, as well as the length of each button
- a convincing argument explaining why your design should be chosen