Chapter 2

Fractions

GOAL

You will be able to

- compare and order fractions using a variety of personal strategies
- add and subtract fractions and mixed numbers using models, drawings, and symbols
- solve problems that involve adding and subtracting fractions
- estimate sums and differences of fractions and mixed numbers
- communicate about estimation strategies

Medicine wheels are symbols of creation and the cycles of life. Suppose that the sections of this medicine wheel were equal. What fractions would they show?

Getting Started

YOU WILL NEED

- pattern blocks
- triangle dot paper

Sweatshirt Sales

The members of a track team are selling sweatshirts to raise money so they can go to an out-of-town meet.

How many different fractions or mixed numbers can you use to represent the sweatshirts?

A. What is true about $\frac{6}{12}$ of the sweatshirts?

B. Why does $\frac{1}{2}$ also describe the sweatshirts in part A?

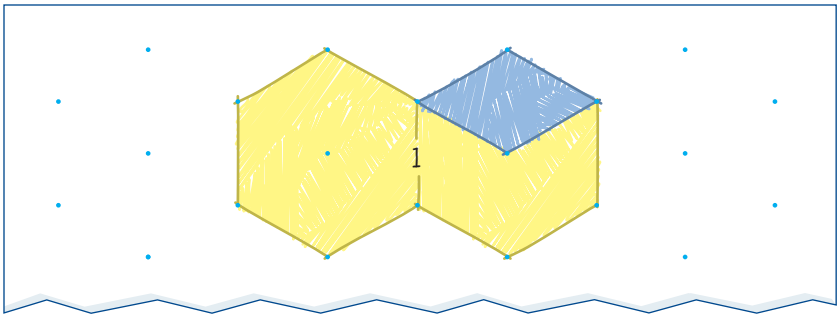
C. What is true about $\frac{1}{3}$ of the sweatshirts?

D. What other fraction describes the sweatshirts in part C?

E. Joe said that $\frac{3}{2}$ of the tables are being used. Explain why he said this.

F. Describe the picture on page 42 using at least five other fractions. Explain what each fraction represents.

G. Suppose that a double hexagon represents 1. You could represent $\frac{1}{6}$ with a blue rhombus, since 6 blue rhombuses would cover the double hexagon.



Model each of your fractions in part F, using a double hexagon to represent 1. Draw your models on dot paper.

What Do You Think?

Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

1. There are equivalent fractions for $\frac{2}{3}$ and $\frac{1}{4}$ that have the same denominator.

2. The fraction $\frac{6}{8}$ is in lowest terms.

3. This shape could represent $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{2}{5}$.

4. When you add two fractions, the sum is always less than 1.

2.1 Comparing Fractions

YOU WILL NEED
- coloured pencils
- a number line

GOAL

Compare and order fractions using benchmarks and equivalent fractions.

LEARN ABOUT the Math

Sarah's math teacher has a new way to decide which students will present their projects first. In groups of 10, each student pulls a slip of paper from a jar. A fraction or **mixed number** is written on each slip. The student with the fourth greatest number will present first.

In Sarah's group, the slips were

$2\frac{1}{3}$ $\frac{8}{9}$ $\frac{12}{5}$ $\frac{2}{9}$ $\frac{15}{18}$ $\frac{7}{9}$ $\frac{2}{3}$ $\frac{4}{6}$ $\frac{2}{5}$ $\frac{4}{5}$

Sarah chose $\frac{8}{9}$.

The students decided to place the fractions on a number line to help them see the order.

lowest terms

an equivalent form of a fraction with a numerator and a denominator that have no common factors other than 1; for example, $\frac{3}{4}$ is the lowest term form of $\frac{12}{16}$, since $\frac{3}{4} = \frac{12}{16}$, and 3 and 4 have no common factors other than 1

common denominator

a common multiple of two or more denominators; for example, a common denominator for $\frac{2}{3}$ and $\frac{3}{6}$ would be any multiple of 6. If you use the least common multiple of the denominators, the common denominator is called the least common denominator

Will Sarah present first in her group?

A. How do you know that $\frac{2}{5}$ is to the left of $\frac{4}{5}$ on the number line?

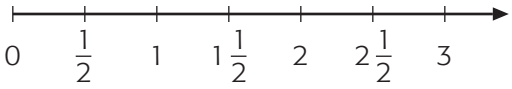
B. How would you decide where $\frac{2}{9}$ goes?

C. How would writing $\frac{4}{6}$ in **lowest terms** help you place it on the number line?

D. How would renaming $\frac{12}{5}$ as a mixed number help you place it on the number line?

E. How would writing $\frac{4}{6}$ and $\frac{15}{18}$ as equivalent fractions with a **common denominator** help you place $\frac{15}{18}$ on the number line?

F. Place all the fractions from Sarah's group on the number line.



G. Which fraction is fourth greatest? Will Sarah present first in her group?

Reflecting

H. Before you placed the numbers on the number line, how might you have known that the student holding $\frac{2}{9}$ or $\frac{2}{5}$ had no chance of presenting first?

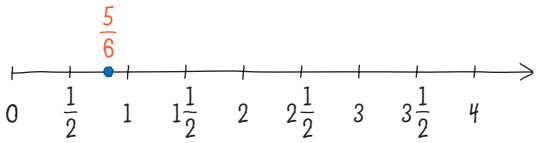
I. What strategies did you use to place the numbers on the number line?

WORK WITH the Math

Example 1 | Ordering numbers on a number line

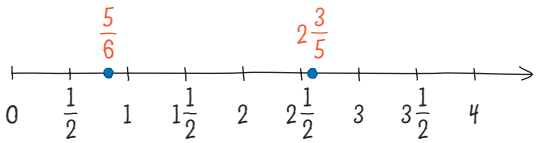
Place these numbers on a number line: $\frac{5}{6}$, $2\frac{3}{5}$, $\frac{31}{8}$, $\frac{2}{8}$, $\frac{3}{9}$.

Sarah's Solution

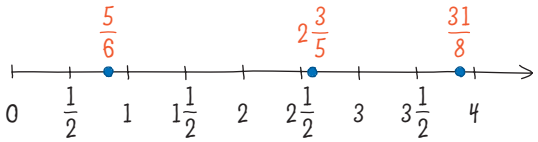


I went through the numbers from left to right.

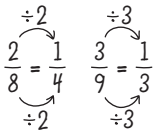
I know that $\frac{5}{6}$ is more than $\frac{1}{2}$ but less than 1.



I know that $2\frac{3}{5}$ is a bit more than $2\frac{1}{2}$, since $\frac{3}{6} = \frac{1}{2}$ and $\frac{3}{5} > \frac{3}{6}$. I know this since each fifth is more than each sixth.

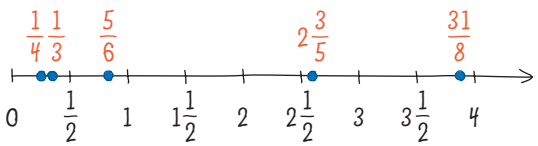


I know that $\frac{32}{8} = 4$, so $\frac{31}{8}$ is a little less than 4.



I divided $\frac{2}{8}$ and $\frac{3}{9}$ by common factors to rename them in lower terms.

I know that 1 fourth piece is less than 1 third piece, so $\frac{1}{4} < \frac{1}{3}$.

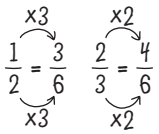


I know that both $\frac{1}{4}$ and $\frac{1}{3}$ are less than $\frac{1}{2}$, since 1 piece out of 4 or 1 piece out of 3 is less than 1 piece out of 2.

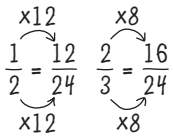
Example 2 | Locating a fraction between fractions

Name some fractions that are between $\frac{1}{2}$ and $\frac{2}{3}$.

Ryan's Solution



I renamed $\frac{1}{2}$ and $\frac{2}{3}$ using a common denominator. I couldn't think of a fraction between $\frac{3}{6}$ and $\frac{4}{6}$, so I used equivalent fractions with a common denominator of 24 so that the numerators were farther apart.



Some fractions between $\frac{1}{2}$ and $\frac{2}{3}$ are $\frac{13}{24}$, $\frac{14}{24}$ or $\frac{7}{12}$, and $\frac{15}{24}$ or $\frac{5}{8}$.

A Checking

1. Write each pair of fractions as equivalent fractions with a common denominator.

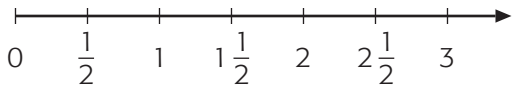
a) $\frac{3}{5}$ and $\frac{2}{4}$

b) $\frac{5}{8}$ and $\frac{3}{4}$

c) $\frac{2}{10}$ and $\frac{1}{15}$

d) $\frac{2}{3}$ and $\frac{1}{8}$

2. a) Place $1\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{7}{5}$ on the number line.



b) List the fractions in order from least to greatest.

3. Compare each pair of fractions using a strategy of your choice.

a) $\frac{3}{7}$ and $\frac{2}{3}$

b) $\frac{2}{5}$ and $\frac{1}{2}$

c) $\frac{8}{6}$ and $\frac{4}{8}$

B Practising

4. Rewrite each fraction in lowest terms.

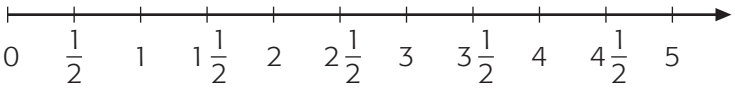
a) $\frac{4}{8}$

b) $\frac{10}{15}$

c) $\frac{15}{6}$

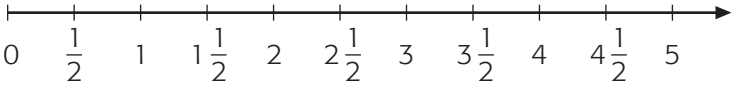
d) $\frac{10}{6}$

5. a) Place $\frac{15}{4}$, $2\frac{2}{5}$, $\frac{34}{10}$, $\frac{5}{8}$, and $\frac{6}{9}$ on the number line.



b) List the fractions in order from least to greatest.

6. a) Place $2\frac{2}{5}$, $3\frac{1}{2}$, $\frac{8}{7}$, $\frac{7}{8}$, and $\frac{4}{5}$ on the number line.



b) List the fractions in order from greatest to least.

7. Compare each pair of fractions using different strategies.

a) $\frac{4}{9}$ and $\frac{5}{6}$ **b)** $\frac{4}{5}$ and $\frac{1}{6}$ **c)** $\frac{8}{3}$ and $\frac{13}{15}$

8. Which number in each list is out of order?

a) $\frac{1}{6}, \frac{2}{5}, \frac{4}{9}, \frac{3}{8}, \frac{9}{5}$

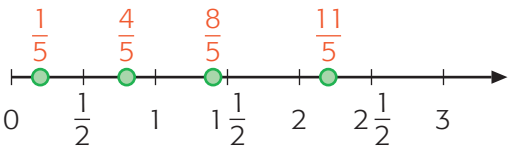
b) $\frac{12}{5}, \frac{11}{3}, 2\frac{1}{2}, \frac{11}{4}, \frac{11}{2}$

c) $\frac{1}{10}, \frac{4}{7}, \frac{7}{6}, \frac{2}{3}, \frac{8}{5}$

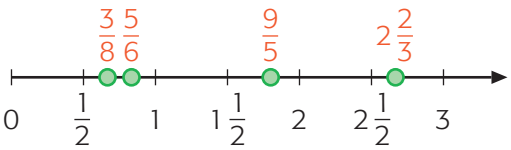
d) $\frac{3}{4}, \frac{2}{10}, \frac{11}{12}, \frac{6}{5}, \frac{3}{2}$

9. Which fraction is in the wrong location?

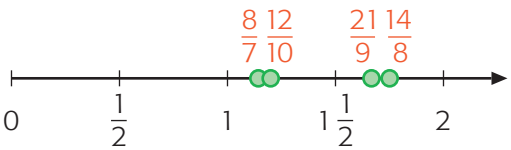
a)



b)



c)



10. On which quiz did Jane do best?

Quiz	A	B	C
Score	$\frac{31}{40}$	$\frac{25}{30}$	$\frac{20}{25}$

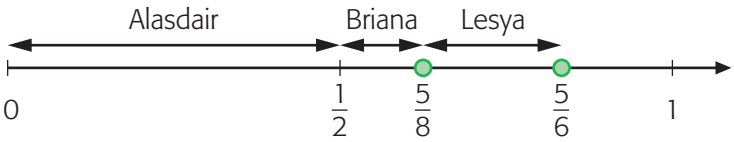
Reading Strategy

Look for the important information in this problem.
Record your ideas.

11. Mike's test marks kept going up by 2, but so did the total possible score on the tests. Were his marks improving? Explain.

12. Alasdair, Briana, and Lesya played a series of chess games. They reported what fraction of the games they won:

- Alasdair said that he won less than $\frac{1}{2}$ of his games.
- Briana said that she won between $\frac{1}{2}$ and $\frac{5}{8}$ of her games.
- Lesya said that she won between $\frac{5}{8}$ and $\frac{5}{6}$ of her games.



Name two possible fractions for each student.

13. Choose two fractions in which the numerators and denominators are both more than 2 apart; for example, $\frac{3}{5}$ and $\frac{7}{10}$.

- Create a new fraction by using a numerator between the two numerators and a denominator between the two denominators; for example, $\frac{5}{8}$.
- How does the new fraction compare with the original two fractions?
- Try some more examples. Does this result always seem to be true?

14. How can you tell whether a fraction is greater than $\frac{1}{2}$?

15. Why is it easier to compare $\frac{2}{3}$ with $\frac{2}{7}$ than it is to compare $\frac{2}{3}$ with $\frac{4}{7}$ using mental strategies?

2.2 Exploring Adding and Subtracting Fractions with the Same Denominator

YOU WILL NEED

- coloured tiles
- grid paper
- pencil crayons

GOAL

Describe fraction addition and subtraction models with equations.

EXPLORE the Math

Nayana filled a 3×4 grid with coloured tiles. She wrote two fraction equations to describe how the different colours filled the space.

$\frac{2}{12} + \frac{10}{12} = \frac{12}{12}$ and $\frac{12}{12} - \frac{2}{12} = \frac{10}{12}$

What fraction equations can you write to describe a grid covered with coloured tiles?

MATH GAME

Super Sixes

Number of players: 2 to 4

YOU WILL NEED
- 2 dice

How to Play

1. Roll the dice twice to create two pairs of numbers.
2. Add the numbers in one pair. Multiply the numbers in the other pair.
3. Calculate the least common multiple (LCM) of the two results.
4. Score 1 point if you calculate the LCM correctly.
5. Score 1 more point if one digit of your LCM is 6.
6. Score 2 more points if your LCM is a multiple of 6.
7. The first player to reach 12 points wins.

Jacob's Turn

I rolled a 3 and a 5. Then I rolled a 2 and a 2.

I decided to multiply 3 and 5, and add 2 and 2.

$3 \times 5 = 15$ $\quad\quad$ $2 + 2 = 4$

15, 30, 45, 60

The LCM for 15 and 4 is 60, since 60 is the first multiple of 15 with 4 as a factor.

I get 4 points:
- 1 point for calculating the 60 correctly
- 1 point for the 6 in 60
- 2 points since 60 is a multiple of 6

2.3 Adding Fractions with Fraction Strips

YOU WILL NEED

- Fraction Strips (Blackline Master)

GOAL

Add fractions less than 1 using fraction strips.

LEARN ABOUT the Math

Denis is reading a book. Last weekend, he read $\frac{1}{3}$ of the book. Yesterday, he read $\frac{1}{4}$ more of it.

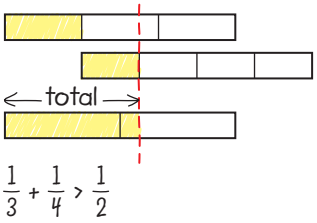
What fraction of the book has Denis read?

Example 1 | Estimating sums using fraction strips

Estimate $\frac{1}{3} + \frac{1}{4}$ using fraction strips.

Denis's Solution

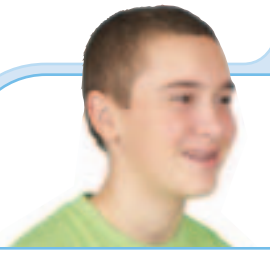
I modelled the two fractions using fraction strips.



First, I made a $\frac{1}{3}$ strip and a $\frac{1}{4}$ strip.

To add, I put the $\frac{1}{4}$ strip at the end of the $\frac{1}{3}$ strip.

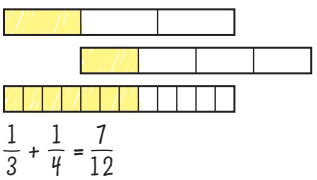
I compared the total with the $\frac{1}{2}$ strip. The sum is a bit more than $\frac{1}{2}$.



Example 2 | Adding using fraction strips

Add $\frac{1}{3} + \frac{1}{4}$ using fraction strips.

Jacob's Solution



First, I added a $\frac{1}{4}$ strip to the end of a $\frac{1}{3}$ strip.

Then, I looked for a strip to match the total length. Since12 is a common multiple of 3 and 4, I looked for a 12ths strip.

$\frac{7}{12}$ is as long as $\frac{1}{3}$ and $\frac{1}{4}$ together.

Since $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$, it makes sense that the $\frac{7}{12}$ strip matched the total length.

Reflecting

A. How does Denis's estimate show that Jacob's answer is probably correct?

B. How could you have predicted that Jacob's answer might involve a 12ths strip?

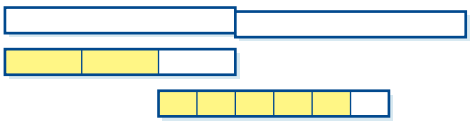
WORK WITH the Math

Example 3 | Adding using models

Estimate and then add $\frac{2}{3} + \frac{5}{6}$.

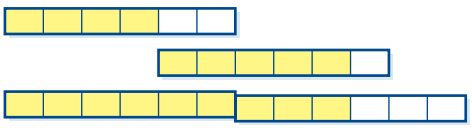
Solution

2 whole fraction strips:



$\frac{2}{3} + \frac{5}{6} > 1$ $\frac{2}{3} + \frac{5}{6} < 2$

Estimate: The answer is more than 1, since the shaded part is more than 1 whole strip. The shaded part is less than 2 whole strips, however. It's about $1\frac{1}{2}$.



To add $\frac{2}{3}$ and $\frac{5}{6}$, use strips with a common denominator of 6.

$\frac{4}{6} + \frac{5}{6} = \frac{9}{6}$

$\frac{2}{3} = \frac{4}{6}$

$\frac{9}{6} = 1\frac{3}{6}$

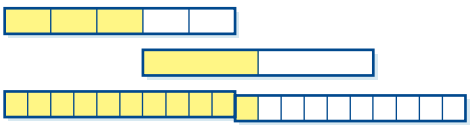
$\frac{2}{3} + \frac{5}{6} = 1\frac{3}{6}$

Since $\frac{3}{6} = \frac{1}{2}$, $1\frac{1}{2}$ was a good estimate for $\frac{2}{3} + \frac{5}{6}$.

A Checking

1. Write the addition that each model represents.

a)



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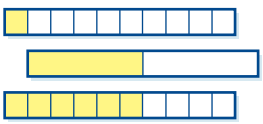


2. a) How do you know that $\frac{3}{4} + \frac{1}{6} < 1$?

b) Calculate $\frac{3}{4} + \frac{1}{6}$ using fraction strips. Show your work.

B Practising

3. Estimate. Show your work.

a) $\frac{2}{3} + \frac{1}{10}$ **b)** $\frac{1}{4} + \frac{9}{10}$ **c)** $\frac{2}{3} + \frac{1}{2}$ **d)** $\frac{5}{6} + \frac{3}{4}$

4. Calculate.

a) $\frac{3}{5} + \frac{1}{5}$ **c)** $\frac{1}{6} + \frac{1}{4}$ **e)** $\frac{5}{6} + \frac{1}{3}$

b) $\frac{2}{3} + \frac{2}{3}$ **d)** $\frac{1}{3} + \frac{7}{12}$ **f)** $\frac{5}{6} + \frac{1}{4}$

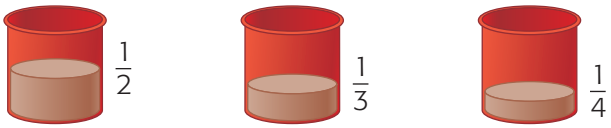
5. Yesterday, Jacques read $\frac{1}{3}$ of the novel *Les beaux jours*. Today, he read $\frac{1}{6}$ of the novel. Use a fraction to describe how much of the novel Jacques has read so far.

6. Francis added fractions with different denominators using fraction strips. His total was one whole strip. List six pairs of fractions he might have been adding.

7. Abby watched one television program for $\frac{1}{4}$ of an hour and then watched another program for 20 min. For what fraction of an hour did Abby watch television?

8. A fraction with a denominator of 4 is added to a fraction with a denominator of 6. What denominator might the answer have? Explain.

9. Yan poured sand into three identical pails. Will all the sand fit in one of these pails? Explain.



10. When you add any two counting numbers (such as 1, 2, 3, …), the answer is always greater than either number. Is the same true when you add any two fractions? Explain.

11. Why is it quicker to add $\frac{5}{12}$ and $\frac{11}{12}$ than to add $\frac{5}{12}$ and $\frac{3}{4}$?

2.4 Subtracting Fractions with Fraction Strips

YOU WILL NEED

- Fraction Strips (Blackline Master)

GOAL

Subtract fractions less than 1 using fraction strips.

LEARN ABOUT the Math

The student council will make a profit on a dance if $\frac{1}{4}$ of the students buy tickets. So far, only $\frac{1}{6}$ of the students have bought tickets.

What fraction of the students still need to buy tickets for the student council to make a profit?

A. Is the fraction probably greater than $\frac{1}{10}$ or less than $\frac{1}{10}$? Explain.

B. How can you use fraction strips to model the problem?

C. What fraction of the students still need to buy tickets for the student council to make a profit?

Reflecting

D. What strategy did you use to answer part A?

E. Why might you subtract two fractions to answer part C?

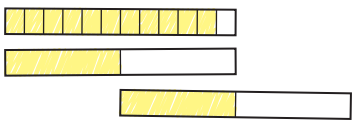
F. Explain how you can use fraction strips to subtract a different pair of fractions, such as $\frac{2}{3} - \frac{1}{4}$.

WORK WITH the Math

Example 1 | Subtracting from a fraction less than 1

Estimate and then subtract $\frac{11}{12} - \frac{1}{2}$.

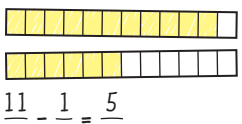
Ryan's Solution



$\frac{11}{12} - \frac{1}{2}$ looks like almost $\frac{1}{2}$.

I modelled both fractions.

The greater fraction is greater by about $\frac{1}{2}$.



$\frac{11}{12} - \frac{1}{2} = \frac{5}{12}$

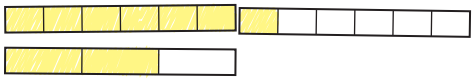
Then, I modelled equivalent fractions with the same denominator. I used $\frac{1}{2} = \frac{6}{12}$.

The $\frac{11}{12}$ strip is $\frac{5}{12}$ longer than the $\frac{6}{12}$ strip. Since $\frac{5}{12}$ is close to $\frac{1}{2}$, the answer makes sense.

Example 2 | Subtracting from a fraction greater than 1

Subtract $\frac{2}{3}$ from $\frac{7}{6}$.

Megan's Solution



$\frac{7}{6} - \frac{2}{3} = \frac{3}{6}$

I modelled both fractions. To model $\frac{7}{6}$, I used a $\frac{6}{6}$ strip and another $\frac{1}{6}$ strip.

When I put the $\frac{2}{3}$ strip next to the $\frac{7}{6}$ strip, I saw that $\frac{7}{6}$ is $\frac{3}{6}$ longer.

I can also write the difference as $\frac{1}{2}$, since I can write $\frac{3}{6}$ in lowest terms as $\frac{1}{2}$.

A Checking

1. a) Why is $\frac{4}{5} - \frac{2}{5}$ equal to $\frac{2}{5}$?

b) Estimate $\frac{4}{5} - \frac{1}{3}$.

c) Calculate $\frac{4}{5} - \frac{1}{2}$ using fraction strips.

2. Suppose that $\frac{3}{4}$ of the students in your class have pets and that $\frac{1}{6}$ have more than one pet. Calculate the fraction of the students with only one pet.

B Practising

3. Calculate.

a) $\frac{5}{6} - \frac{2}{6}$ **b)** $\frac{5}{8} - \frac{2}{8}$ **c)** $\frac{5}{12} - \frac{2}{12}$

4. What pattern do you notice in question 3? Why does this pattern make sense?

5. Estimate each difference.

a) $\frac{7}{12} - \frac{1}{5}$ **b)** $\frac{7}{10} - \frac{1}{4}$ **c)** $\frac{11}{12} - \frac{1}{5}$

6. Calculate.

a) $\frac{3}{5} - \frac{1}{10}$ **c)** $\frac{8}{3} - \frac{3}{4}$ **e)** $\frac{3}{10} - \frac{1}{5}$

b) $\frac{5}{2} - \frac{9}{12}$ **d)** $\frac{7}{4} - \frac{4}{6}$ **f)** $\frac{5}{4} - \frac{5}{12}$

7. a) Draw a shape. Colour the shape so that $\frac{2}{3}$ is blue and $\frac{1}{6}$ is yellow.

b) What fraction tells how much more of the total shape is blue than yellow?

c) What fraction tells how much is neither blue nor yellow?

8. Rosa wrote $\frac{1}{2}$ of her book report on Tuesday and another $\frac{1}{5}$ on Wednesday.

a) What fraction of her book report does she still have left to write?

b) How do you know that your answer makes sense?

9. Aiden said that he calculated $\frac{3}{4} - \frac{2}{3}$ by calculating $1 - \frac{2}{3}$ and then subtracting $\frac{1}{4}$. Do you agree with what he did? Why or why not?

10. At her French school, Myriam surveyed students about their favourite activities. Complete each fraction.

Activity	Fraction of students who prefer activity
radio étudiante	$\frac{1}{4}$
jazz band	$\frac{1}{6}$
journal étudiant	$\frac{1}{12}$
art dramatique	$\frac{1}{3}$

a) $\frac{\square}{\square}$ more students prefer art dramatique to radio étudiante

b) $\frac{\square}{\square}$ more students prefer radio étudiante to journal étudiant

c) $\frac{\square}{\square}$ more students prefer jazz band to journal étudiant

11. To calculate $\frac{4}{3} - \frac{3}{4}$, Ann adds $\frac{1}{4}$ to $\frac{1}{3}$.

a) Model $\frac{4}{3}$ and $\frac{3}{4}$.

b) Explain Ann's method.

c) What is $\frac{4}{3} - \frac{3}{4}$?

12. The Labrador block in the Canadian Quilt of Belonging is shown below.

a) Estimate what fraction of the block is green.

b) Estimate what fraction of the block is grey.

c) About how much more of the block is green than grey?

13. The Ukrainian Bilingual School is holding a talent show. Between $\frac{1}{4}$ and $\frac{1}{2}$ of the performers will dance. At least $\frac{1}{2}$ will read poetry. The rest will play music. What fraction of the performers will play music? Explain your thinking.

14. **a)** Choose two fractions. Model them with fraction strips.

b) Add your fractions.

c) Subtract one fraction from the other.

d) Is the denominator of the sum the same as the denominator of the difference? Explain.

15. How is subtracting fractions like adding them?

2.5 Exploring Fraction Addition on Grids

YOU WILL NEED

- grid paper
- counters
- chart paper

GOAL

Add fractions with grids and counters.

EXPLORE the Math

Denis served a tray of spring rolls.
Jacob ate $\frac{1}{3}$ of the spring rolls, and Ryan ate $\frac{2}{5}$ of them.

How can you calculate, using a grid and counters, what fraction of the spring rolls Jacob and Ryan ate?

2.6 Subtracting Fractions with Grids

YOU WILL NEED

- grid paper
- counters

GOAL

Subtract fractions concretely.

LEARN ABOUT the Math

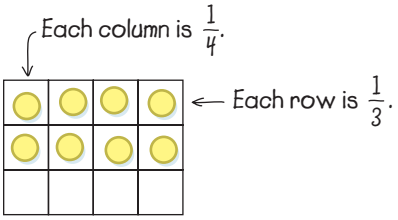
Ryan is awake for $\frac{2}{3}$ of every day. He spends $\frac{1}{4}$ of every school day either at school or on the bus.

What fraction of a school day is left for other activities?

Example 1 | Subtracting with grid paper

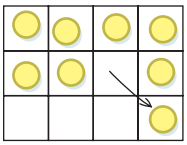
Subtract $\frac{2}{3} - \frac{1}{4}$ using a grid and counters.

Oshana's Solution

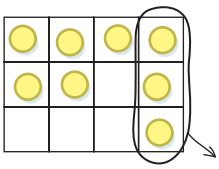


I used a 3×4 grid, so I could easily show thirds and fourths.

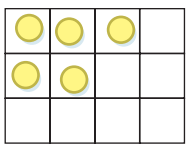
To show $\frac{2}{3}$, I filled in two rows of the grid.



$\frac{1}{4}$ is one column, so I knew that I'd need to remove counters from one whole column. I moved a counter to fill a column.



I removed all the counters in this column and counted how many counters were left. There were 5 counters left.



There are 12 sections in the grid. Each section is $\frac{1}{12}$.

$\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

Since there were 5 counters left, $\frac{5}{12}$ of a school day is left for other activities.

Reflecting

A. How did Oshana's grid give her equivalent fractions for $\frac{2}{3}$ and $\frac{1}{4}$ with a common denominator?

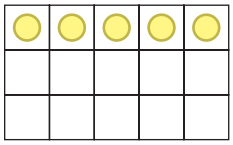
B. What size of grid would you use to calculate $\frac{3}{4} - \frac{1}{2}$? Explain how you would use counters to model the subtraction.

WORK WITH the Math

Example 2 | Subtracting from a whole

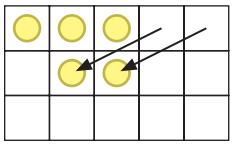
Darby mowed $\frac{1}{3}$ of a lawn before lunch and another $\frac{2}{5}$ after lunch. How much of the lawn is left to mow?

Solution



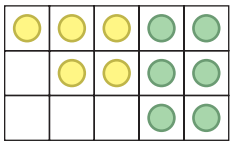
Use a 3×5 grid to show thirds and fifths.

Model $\frac{1}{3}$ using one row of counters.



Now prepare to model adding $\frac{2}{5}$.

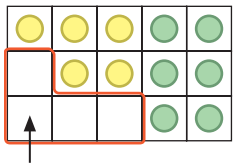
Each column is $\frac{1}{5}$, so clear 2 columns to make room for counters to be added.
Move 2 counters.



$$\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$$

Now add 2 columns of counters.

That adds $\frac{2}{5}$. There are 11 counters, so $\frac{11}{15}$ of the lawn has been mowed.



The amount left to mow

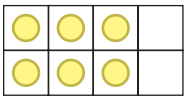
$$1 - \left(\frac{1}{3} + \frac{2}{5}\right) = \frac{4}{15}$$

Now subtract. Since 4 sections do not have counters, $\frac{4}{15}$ of the lawn is left to mow.

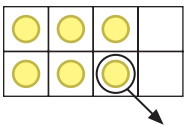
Example 3 | Determining what was removed

There was $\frac{3}{4}$ of a pie left, and Dan ate some of it. After he finished, $\frac{5}{8}$ of the pie was left. How much of the pie did Dan eat?

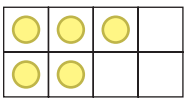
Solution



You could use a 4×8 grid, but fourths and eighths can also be shown on a 2×4 grid.
Each square is $\frac{1}{8}$ of the grid.
Each column is $\frac{1}{4}$.
Three columns of counters are $\frac{3}{4}$, or $\frac{6}{8}$, of the grid.



To have $\frac{5}{8}$ left, remove 1 of the 6 counters. Subtract $\frac{1}{8}$.



Each counter represents $\frac{1}{8}$ of the pie.
Since 1 counter was taken away, Dan ate $\frac{1}{8}$ of the pie.

$\frac{3}{4} - \frac{\blacksquare}{\blacksquare} = \frac{5}{8}$

$\frac{6}{8} - \frac{\blacksquare}{\blacksquare} = \frac{5}{8}$

$\frac{\blacksquare}{\blacksquare} = \frac{1}{8}$

A Checking

1. Calculate each difference using a grid and counters. Show your work.

a) $\frac{2}{3} - \frac{1}{5}$

b) $\frac{5}{6} - \frac{1}{4}$

2. $\frac{7}{12}$ of the flowers in a garden have bloomed, and $\frac{1}{3}$ of these flowers are geraniums. Use a model to show what fraction of the flowers that have bloomed are other flowers.

B Practising

3. Calculate each difference. Show your work.

a) $\frac{4}{5} - \frac{2}{3}$ **c)** $\frac{1}{3} - \frac{2}{7}$ **e)** $\frac{3}{5} - \frac{1}{4}$

b) $\frac{1}{3} - \frac{1}{4}$ **d)** $\frac{7}{8} - \frac{2}{3}$ **f)** $\frac{3}{4} - \frac{2}{5}$

4. Ella phoned $\frac{1}{3}$ of the track team members on her list last weekend. She phoned $\frac{1}{5}$ of the members on Saturday.

a) Did she phone more team members on Saturday or on Sunday? How do you know?

b) What fraction of the team did she phone on Sunday?

5. André and his mom drove from Saint-Norbert to Brandon and back. When they left, the gas tank was $\frac{3}{4}$ full. When they returned, the gas tank was $\frac{1}{8}$ full. What fraction of a tank of gas did they use on the trip? How do you know that your answer is reasonable?

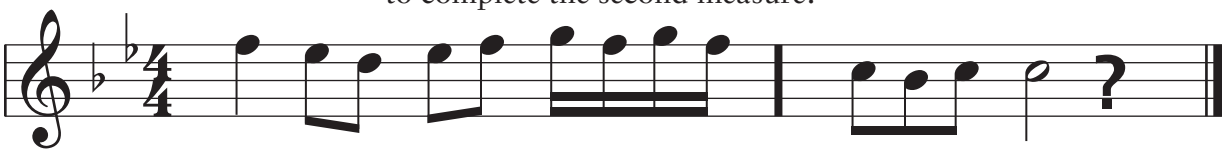
6. Make up your own problem that is similar to question 5, and then solve it.

7. Suppose that you subtract one fraction between $\frac{1}{2}$ and 1 from another fraction between $\frac{1}{2}$ and 1. Is each statement always true, sometimes true, or never true? Explain your thinking.

A. The difference is less than $\frac{1}{2}$.

B. The difference is greater than $\frac{1}{4}$.

8. These musical notes are like fractions. The total of the fractions in each measure is 1. What notes can you add to complete the second measure?



$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$ $\qquad$ $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{2} + ?$

9. Why might it be easier to subtract fractions with a grid and counters than with fraction strips?

MATH GAME

Fraction Tic-Tac-Toe

In this game, you will use unit fractions to play tic-tac-toe.

Number of players: 2

YOU WILL NEED

- 9 square index cards
- 2 dice
- 2 colours of counters

How to Play

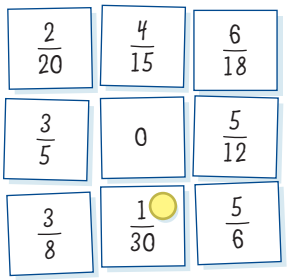
1. Place nine cards on a table to form a square.
   Write "0" on the middle card.
   Write fractions on the other cards.

For numerators, use	For denominators, use
1, 2, 3, 4, 5, or 6	2, 3, 4, 5, 6, 8, 10, 12, 15, 18, 20, or 30

2. Roll the dice. Use the numbers you roll as the denominators of two fractions. Use 1 as the numerator of the two fractions.
3. If the sum or difference of your two fractions is on a card, put one of your counters on the card.
4. Take turns rolling and calculating. Check each other's work.
5. The winner is the first player who has three counters in a horizontal, vertical, or diagonal line.

Oshana's Turn

These were our cards. I rolled a 6 and a 5.
The difference between $\frac{1}{5}$ and $\frac{1}{6}$ is $\frac{1}{30}$,
so I put a counter on that card.

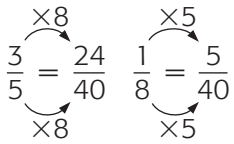


Mid-Chapter Review

Frequently Asked Questions

Q: **How do you compare two fractions, such as $\frac{3}{5}$ and $\frac{1}{8}$?**

A1: You can compare each fraction to a benchmark, such as $\frac{1}{2}$. Since $\frac{3}{5}$ is greater than $\frac{1}{2}$ and $\frac{1}{8}$ is less than $\frac{1}{2}$, $\frac{3}{5} > \frac{1}{8}$.

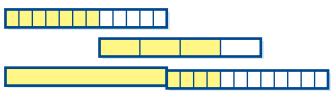


A2: You can use equivalent fractions with a common denominator. The least common multiple of 5 and 8 is 40. Rename the fractions. $\frac{24}{40} > \frac{5}{40}$, so $\frac{3}{5} > \frac{1}{8}$.

Q: **Why is it easier to add or subtract fractions when the denominators are the same?**

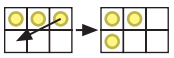
A: When the denominators are the same, then all the pieces are the same size. You can just add or subtract the numerators to count the pieces.

Q: **What models are helpful for adding and subtracting fractions?**

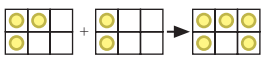


A1: You can use fraction strips that show different numbers of sections. For example, $\frac{7}{12} + \frac{3}{4} = 1\frac{4}{12}$.

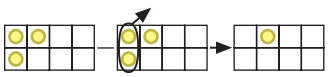
A2: You can use a grid and counters. Use the denominators of the fractions to decide what size of grid to use. For example, for $\frac{1}{2} + \frac{1}{3}$, use a 2×3 grid.



Start with $\frac{1}{2}$. Move 1 counter so you can add $\frac{1}{3}$.



Add $\frac{1}{3}$. There are 5 counters, so $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.



For $\frac{3}{8} - \frac{1}{4}$, use a 2×4 grid or a 4×8 grid.

$\frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

Practice

Lesson 2.1

1. Write each pair of fractions as equivalent fractions with a common denominator.

a) $\frac{3}{5}$ and $\frac{2}{6}$ **b)** $\frac{3}{5}$ and $\frac{2}{10}$ **c)** $\frac{3}{10}$ and $\frac{4}{15}$

2. Write an equivalent fraction in lower terms.

a) $\frac{4}{6}$ **b)** $\frac{12}{18}$ **c)** $\frac{21}{15}$

3. Order from least to greatest: $3\frac{1}{4}, \frac{5}{6}, \frac{1}{9}, \frac{2}{3}, \frac{8}{5}, \frac{7}{3}$.

Lesson 2.3

4. Estimate each sum and then calculate. Show your work.

a) $\frac{3}{5} + \frac{4}{5}$ **b)** $\frac{7}{12} + \frac{1}{2}$ **c)** $\frac{7}{10} + \frac{2}{5}$ **d)** $\frac{2}{3} + \frac{11}{12}$

5. At a powwow, $\frac{1}{6}$ of the people were fancy dancers and $\frac{1}{4}$ were traditional ladies. What fraction were fancy dancers or traditional ladies?

Lesson 2.4

6. Calculate.

a) $\frac{4}{10} - \frac{1}{10}$ **b)** $\frac{7}{4} - \frac{5}{12}$ **c)** $\frac{11}{10} - \frac{4}{5}$ **d)** $\frac{3}{4} - \frac{1}{12}$

7. In the Yukon Territory, about $\frac{3}{4}$ of the people are from 15 to 65 years old. About $\frac{1}{5}$ of the people are 14 years old or younger. Use a fraction to describe the difference between the two age groups.

8. Which of these expressions have answers between $\frac{1}{2}$ and $1\frac{1}{2}$? How do you know?

A. $\frac{3}{4} + \frac{1}{5}$ **B.** $\frac{3}{4} + \frac{5}{6}$ **C.** $\frac{3}{8} + \frac{1}{2}$ **D.** $\frac{1}{4} + \frac{2}{3}$

Lesson 2.6

9. What size of grid would you use to model each calculation? Why?

a) $\frac{5}{6} - \frac{1}{2}$ **b)** $1 - \frac{2}{5}$ **c)** $\frac{4}{5} + \frac{2}{3}$ **d)** $\frac{3}{8} + \frac{3}{4}$

2.7 Adding and Subtracting Fractions with Number Lines

YOU WILL NEED
- number lines
- dice

GOAL

Add and subtract fractions using a pictorial model.

LEARN ABOUT the Math

In 2004, about $\frac{2}{3}$ of the Canadian tourists who visited Alberta were from either Saskatchewan or British Columbia. About $\frac{2}{5}$ of Canadian tourists were from British Columbia.

What fraction tells how many more tourists came from British Columbia than from Saskatchewan?

A. Why might you subtract to solve the problem?

B. Think of a number line as a thin fraction strip. How could you use a number line to solve the problem?

C. What fraction tells about how many more tourists were from British Columbia than from Saskatchewan?

Reflecting

D. What subtraction did you complete? How did you show it on a number line?

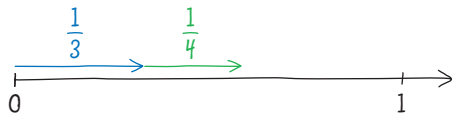
E. How can you check your answer by adding on your number line?

WORK WITH the Math

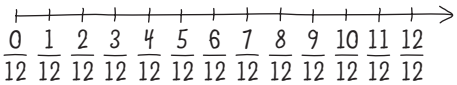
Example 1 | Adding using a number line

Add $\frac{1}{3} + \frac{1}{4}$ using a number line.

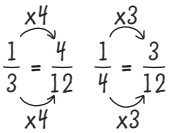
Denis's Solution



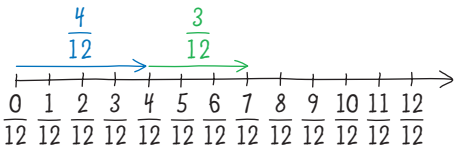
I made a number line and drew $\frac{1}{3}$ and $\frac{1}{4}$.
I could see that the total was a bit more than $\frac{1}{2}$, but I wanted the actual answer.



I used equivalent fractions with a common denominator to make the answer easier to read. Since 12 is the least common denominator for $\frac{1}{3}$ and $\frac{1}{4}$, I used a number line marked in 12ths.



I renamed $\frac{1}{3}$ and $\frac{1}{4}$ in 12ths.



I started the $\frac{3}{12}$ arrow at the end of the $\frac{4}{12}$ arrow to add, just as I did with fraction strips.

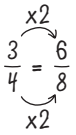
The sum is $\frac{7}{12}$.

$$\frac{1}{3} + \frac{1}{4}$$
$$= \frac{4}{12} + \frac{3}{12}$$
$$= \frac{7}{12}$$

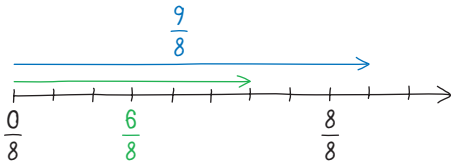
Example 2 | Subtracting using a number line

Calculate $\frac{9}{8} - \frac{3}{4}$ using a number line.

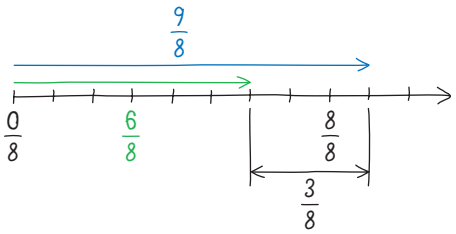
Megan's Solution



The least common multiple of 8 and 4 is 8. I renamed $\frac{3}{4}$ using an equivalent fraction with a denominator of 8.



I knew that $\frac{9}{8}$ is $\frac{1}{8}$ more than 1 $\left(\frac{8}{8}\right)$.
I drew arrows to show $\frac{9}{8}$ and $\frac{6}{8}$.



There are 3 eighths from $\frac{6}{8}$ to $\frac{9}{8}$.

$\frac{9}{8} - \frac{3}{4} = \frac{3}{8}$

A Checking

1. Calculate. Show your work.

a) $\frac{3}{4} + \frac{1}{6}$ **b)** $\frac{6}{5} - \frac{2}{3}$

2. About $\frac{1}{5}$ of the members of the Vancouver Symphony Orchestra play a woodwind. About $\frac{1}{4}$ play the violin.

a) What total fraction of the orchestra do these members represent?

b) What fraction tells how many more members play the violin than a woodwind?

B Practising

3. Calculate.

a) $\frac{2}{3} + \frac{1}{2}$ **b)** $\frac{11}{12} + \frac{1}{4}$ **c)** $\frac{8}{9} - \frac{2}{3}$ **d)** $\frac{6}{7} - \frac{1}{3}$

4. Jake ate $\frac{3}{8}$ of a pan of lasagna, and his dad ate $\frac{1}{4}$ of the pan. Marie and Leah ate the rest. How much lasagna did the girls eat?

5. Leanne put some of her allowance into her bank account to save for a bicycle. After making the deposit, she had $\frac{2}{5}$ of her allowance left. At the end of the week, she still had $\frac{1}{7}$ of her allowance left. What fraction of her allowance did she spend during the week?

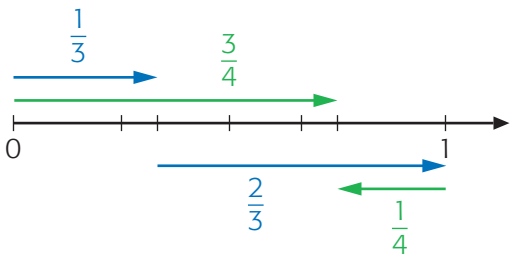
6. A Chinese restaurant makes $\frac{1}{3}$ of its income on Friday and Saturday nights and $\frac{2}{5}$ from lunches during the work week. What fraction of its income is from other meals?

7. Roll a pair of dice twice. Use the numbers you roll to create two fractions.

a) Can you roll numbers so that the sum of the two fractions is $\frac{5}{6}$? Explain.

b) Can you roll numbers so that the difference is $\frac{5}{6}$? Explain.

8. Jarod calculated $\frac{3}{4} - \frac{1}{3}$ using the number line below. How does this number line show that his answer is the same as the answer for $\frac{2}{3} - \frac{1}{4}$?



9. In this chapter, you have added and subtracted fractions with fraction strips, a grid and counters, and a number line. Which method do you prefer? Why?

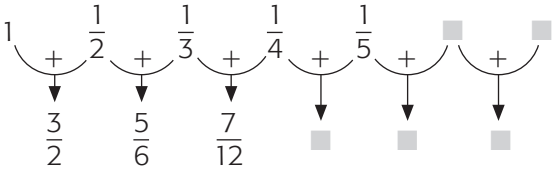
2.8 Exploring Fraction Patterns

GOAL

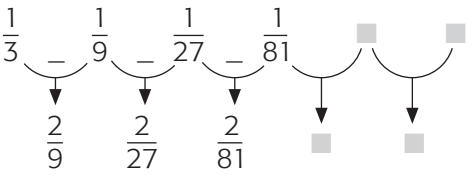
Investigate fraction patterns that involve addition and subtraction.

EXPLORE the Math

Nayana and Jacob are creating fraction patterns.
This is the start of Nayana's pattern.



This is the start of Jacob's pattern.



What patterns can you create using addition or subtraction of fractions?

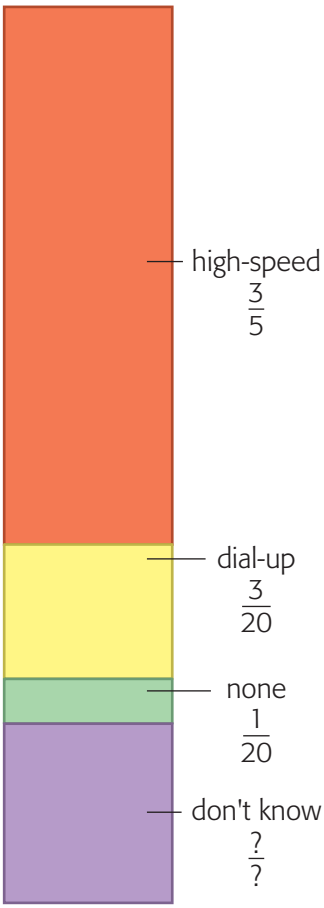
2.9 Adding and Subtracting Fractions

GOAL

Add and subtract fractions less than 1 symbolically.

LEARN ABOUT the Math

Canadian Students' Internet Connections at Home



What kind of Internet connection do you have at home? In 2005, Canadian students from Grades 4 to 11 were asked this question. The graph at the left shows the results.

What fraction of the students said "don't know"?

A. List the fractions that describe the students who said high-speed, dial-up, and none.

B. Calculate each sum.
a) high-speed + dial-up
b) high-speed + dial-up + none

C. What fraction of the students said "don't know"? How do you know?

Reflecting

D. Why is there more than one way to calculate the fraction for "don't know"?

E. Why did you choose the denominator you did in part B to add the fractions?

F. How can you add or subtract two fractions without a model?

WORK WITH the Math

Example 1 | Adding using equivalent fractions

Madeleine's recycling bin is already $\frac{2}{3}$ full. She fills another $\frac{1}{4}$ of the bin. How full is the bin now?

Nayana's Solution

$\frac{2}{3} + \frac{1}{4} = \frac{\square}{\square}$

I had to add the two fractions.

$\frac{2}{3} \overset{\times 4}{=} \frac{8}{12}$ $\frac{1}{4} \overset{\times 3}{=} \frac{3}{12}$

The least common multiple of 3 and 4 is 12.
I used equivalent fractions with 12 as the denominator.

$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

The bin is now $\frac{11}{12}$ full.

Example 2 | Subtracting using equivalent fractions

In Jay's class, $\frac{3}{4}$ of the students were born in Richmond and $\frac{1}{7}$ were born in Surrey. How many more students were born in Richmond than in Surrey?

Jacob's Solution

$\frac{3}{4} - \frac{1}{7} = \frac{\square}{\square}$

I had to subtract the two fractions.

$\frac{3}{4} \overset{\times 7}{=} \frac{21}{28}$ $\frac{1}{7} \overset{\times 4}{=} \frac{4}{28}$

The least common multiple of 4 and 7 is 28.
I used equivalent fractions with 28 as the denominator.

$\frac{21}{28} - \frac{4}{28} = \frac{17}{28}$

If I subtract 4 parts from 21 parts and the parts are all 28ths, there are 17 parts left. In Jay's class, $\frac{17}{28}$ more students were born in Richmond than in Surrey.

A Checking

1. Calculate.

a) $\frac{3}{5} - \frac{1}{5}$ **b)** $\frac{3}{6} + \frac{2}{3}$ **c)** $\frac{7}{8} - \frac{3}{4}$ **d)** $\frac{2}{7} + \frac{2}{3}$

2. At a school party, $\frac{2}{3}$ of the students wore T-shirts and $\frac{1}{5}$ wore long-sleeved shirts. Which fraction is greater? By how much?

B Practising

3. Which of these expressions are equal to $\frac{1}{2}$?

A. $\frac{5}{12} - \frac{1}{3}$ **B.** $\frac{5}{12} + \frac{1}{3}$ **C.** $\frac{3}{7} + \frac{1}{14}$ **D.** $\frac{3}{5} - \frac{1}{10}$

4. In a Grade 7 class, $\frac{1}{5}$ of the students have two pets and $\frac{1}{20}$ have three or more pets.

a) Estimate the fraction of the class with two or more pets.

b) How many students do you think are in the class? Explain.

5. Complete this equation: $\frac{2}{3} + \frac{3}{5} + \frac{\square}{15} = \frac{\square}{15}$.

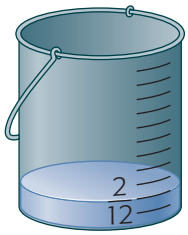
6. Which of these expressions are greater than 1? How do you know?

A. $\frac{2}{3} + \frac{1}{6}$ **B.** $\frac{1}{2} + \frac{3}{5}$ **C.** $\frac{3}{2} - \frac{3}{7}$ **D.** $2 - \frac{3}{4}$

7. Four students added $\frac{3}{4} + \frac{5}{6}$ and got these answers: $\frac{38}{24}$, $1\frac{14}{24}$, $1\frac{7}{12}$, and $\frac{19}{12}$. Are they all correct? How do you know?

8. Calculate using equivalent fractions.

a) $\frac{2}{3} + \frac{3}{7}$ **b)** $\frac{3}{5} + \frac{4}{7}$ **c)** $\frac{3}{4} + \frac{7}{9}$ **d)** $\frac{3}{4} - \frac{1}{3}$



9. Kristen poured water into this pail until it was $\frac{3}{4}$ full. How much water did she add?

10. Two fractions add to $\frac{1}{4}$. Is each statement true or false? Explain.

A. Both fractions are less than $\frac{1}{8}$.

B. One fraction might be $\frac{1}{5}$.

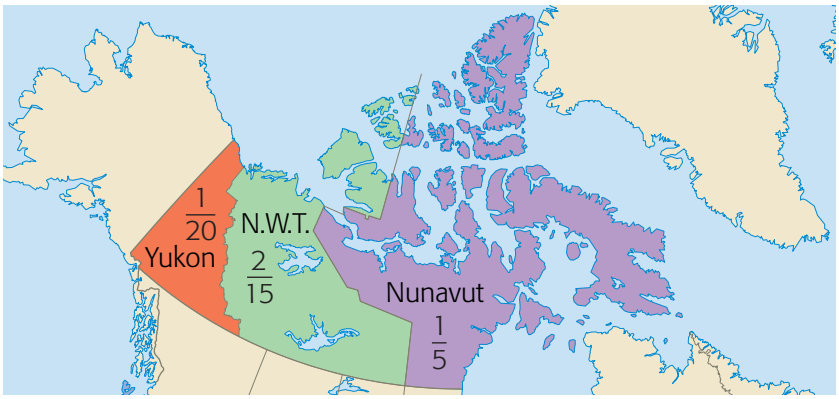
C. One fraction might be $\frac{2}{5}$.

D. The denominators might be 10 and 20.

Reading Strategy

What questions can you ask to help you understand the question?

11. An estimate of the area of each territory is shown as a fraction of Canada's area. About how much of Canada do all three territories cover?



12. Which of these expressions is closest to $\frac{1}{2}$ in value? How close is it?

A. $\frac{3}{4} - \frac{2}{10}$

B. $\frac{4}{5} - \frac{1}{3} + \frac{1}{15}$

C. $\frac{1}{3} + \frac{1}{5} + \frac{1}{10}$

D. $\frac{2}{9} + \frac{1}{6} + \frac{1}{3}$

13. Describe a situation in which you might add $\frac{1}{3} + \frac{1}{4} + \frac{1}{2}$.

How often do you read news online?

- ☐ regularly
- ☐ rarely
- ☐ never

14. About $\frac{1}{3}$ of Canadians read news online regularly. Another $\frac{1}{8}$ read news online rarely. About what fraction of Canadians never read news online?

15. Your friend wants to calculate how much more $\frac{3}{4} + \frac{4}{5}$ is than $\frac{2}{3} + \frac{2}{5}$. How would you explain what to do?

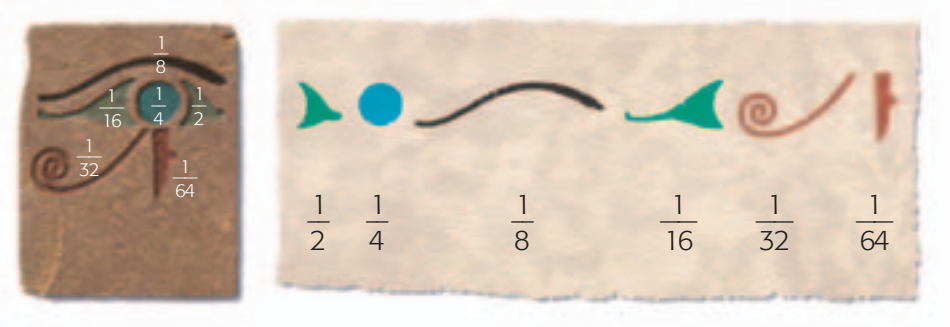
CURIOUS MATH

Egyptian Fractions

The ancient Egyptians used only fractions with a numerator of 1 (called unit fractions).

They used parts of the "eye of Horus" to represent these fractions, as shown below.

They wrote other fractions as sums of the unit fractions.



1. Show that $\frac{2}{3}$ equals $\frac{1}{2} + \frac{1}{6}$.

2. Write each fraction as a sum of unit fractions with different denominators.

a) $\frac{3}{4}$ **b)** $\frac{8}{15}$ **c)** $\frac{19}{24}$

3. Complete the table to show that you can write any unit fraction as the difference of two other unit fractions.

$\frac{1}{3}$	$= \frac{1}{2} - \frac{1}{6}$
$\frac{1}{4}$	$= \frac{1}{\square} - \frac{1}{\square}$
$\frac{1}{5}$	$= \frac{1}{\square} - \frac{1}{\square}$
$\frac{1}{6}$	$= \frac{1}{\square} - \frac{1}{\square}$
$\frac{1}{7}$	$= \frac{1}{\square} - \frac{1}{\square}$
...	...
$\frac{1}{50}$	$= \frac{1}{49} - \frac{1}{2450}$
$\frac{1}{100}$	$= \frac{1}{99} - \frac{1}{9900}$

4. Describe a pattern in the table.

5. Use your pattern to write another fraction as the difference of unit fractions.

2.10 Adding and Subtracting Mixed Numbers

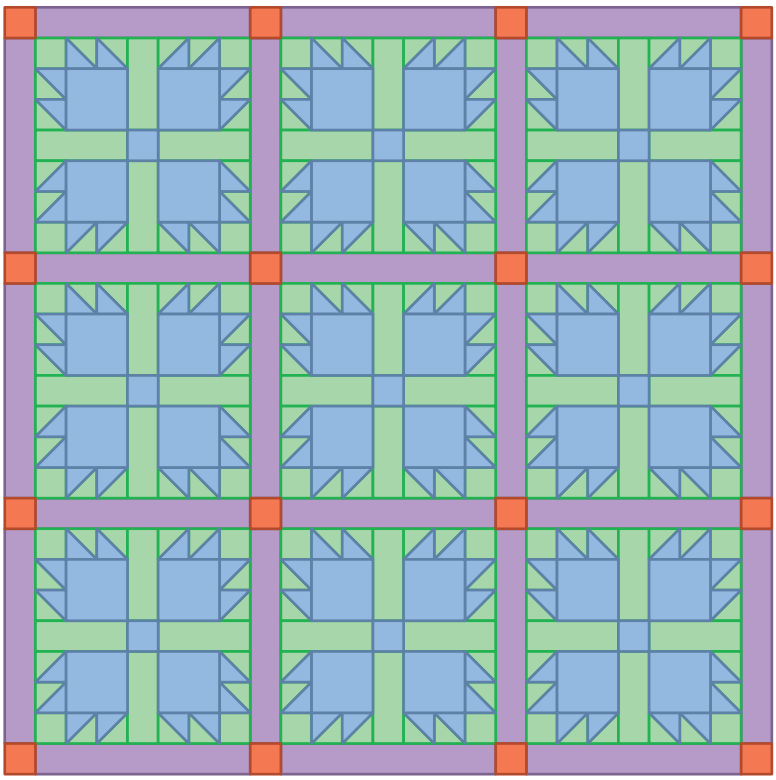
GOAL

Solve problems by adding or subtracting mixed numbers and fractions.

LEARN ABOUT the Math

Oshana is working on a Bear Claw quilt. She has $25\frac{1}{2}$ large blue squares.

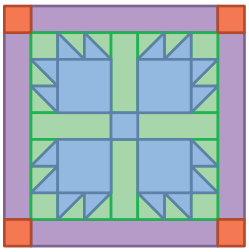
Suppose that Oshana makes 3 quilt sections. How many blue squares will she have left?



Example 1 | Adding wholes and fractions separately

I'll determine how many blue squares are used in each section. Then I'll calculate how many blue squares are in three sections. Then I'll subtract the total from $25\frac{1}{2}$.

Oshana's Solution



This is one section. It has 4 large blue squares ■.

It has 16 small triangles ◺, which can be cut out of 2 large squares.

It has 1 small blue square ■, which is $\frac{1}{4}$ of a large square.

$4 + 2 + \frac{1}{4} = 6\frac{1}{4}$

Each section uses $6\frac{1}{4}$ large blue squares.

$6\frac{1}{4} + 6\frac{1}{4} + 6\frac{1}{4} = 6 + 6 + 6 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

I added to determine how many blue squares are in 3 sections.

$6 + 6 + 6 = 18$

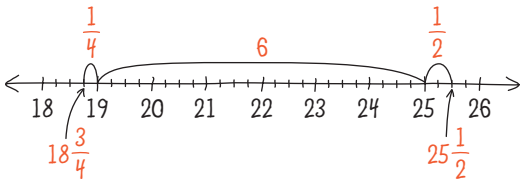
I added the whole numbers.

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

I added the fractions.

$18 + \frac{3}{4} = 18\frac{3}{4}$

I added the two sums.



I had to subtract $18\frac{3}{4}$ from $25\frac{1}{2}$.

I thought about the difference between $18\frac{3}{4}$ and $25\frac{1}{2}$ on a number line.

$\frac{1}{4} + 6 + \frac{1}{2} = 6\frac{3}{4}$

I'll have $6\frac{3}{4}$ squares left.

Example 2 | Calculating using improper fractions

I'll add $6\frac{1}{4}$ three times and subtract the total from $25\frac{1}{2}$.

Ryan's Solution

$6\frac{1}{4} = \frac{24}{4} + \frac{1}{4}$ or $\frac{25}{4}$

I renamed $6\frac{1}{4}$ as an improper fraction.

Each whole is $\frac{4}{4}$, so 6 wholes is $\frac{24}{4}$.

$\frac{25}{4} + \frac{25}{4} + \frac{25}{4} = \frac{75}{4}$

I added $\frac{25}{4}$ three times, once for each section.

$\frac{25}{1} = \frac{100}{4}$ (×4 on both) $\quad \frac{1}{2} = \frac{2}{4}$ (×2 on both)

$\frac{100}{4} + \frac{2}{4} = \frac{102}{4}$

To subtract from $25\frac{1}{2}$, I renamed both the 25 and the $\frac{1}{2}$ as fourths.

$\frac{102}{4} - \frac{75}{4} = \frac{27}{4}$

I subtracted.

$\frac{27}{4} = \frac{24}{4} + \frac{3}{4}$

$= 6\frac{3}{4}$

I renamed the difference as a mixed number.

$\frac{24}{4}$ is 6 wholes. Oshana will have $6\frac{3}{4}$ squares left.

Reflecting

A. If a number line is marked only with whole numbers, why is it easier to estimate a difference using mixed numbers rather than improper fractions?

B. How are the two methods alike? How are they different?

WORK WITH the Math

Example 3 | Adding and subtracting mixed numbers

Caleb mixed $1\frac{1}{2}$ cans of yellow paint with $2\frac{3}{4}$ cans of blue paint.
He used $3\frac{4}{5}$ of these cans to paint a room. How much paint is left?

Solution A: Using mixed numbers

$1\frac{1}{2} = 1 + \frac{1}{2}$ and $2\frac{3}{4} = 2 + \frac{3}{4}$

Add $1\frac{1}{2} + 2\frac{3}{4}$ to determine how much paint Caleb started with.

$1 + 2 = 3$

$$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4}$$
$$= \frac{5}{4}$$

Add the whole numbers, and then add the fractions.

$$3 + \frac{5}{4} = 3 + 1 + \frac{1}{4}$$
$$= 4\frac{1}{4}$$

Add the whole numbers and the fractions.

Caleb started with $4\frac{1}{4}$ cans of paint.

$4\frac{1}{4} - 3\frac{4}{5}$

Subtract to determine how much paint is left.

Since $\frac{4}{5} > \frac{1}{4}$, regroup $4\frac{5}{20}$ to get $3\frac{25}{20}$.

Rename $\frac{1}{4}$ and $\frac{4}{5}$ using a common denominator of 20.

$3 - 3 = 0$

$$\frac{25}{20} - \frac{16}{20} = \frac{9}{20}$$

Subtract the whole numbers, and then subtract the fractions.

There is $\frac{9}{20}$ of a can left.

Solution B: Using improper fractions

$$1\frac{1}{2} = \frac{2}{2} + \frac{1}{2} \quad 2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} \quad 3\frac{4}{5} = \frac{15}{5} + \frac{4}{5}$$
$$= \frac{3}{2} \quad = \frac{11}{4} \quad = \frac{19}{5}$$

Write the mixed numbers as improper fractions.

$$\frac{3}{2} + \frac{11}{4} = \frac{6}{4} + \frac{11}{4}$$
$$= \frac{17}{4}$$

Rename $\frac{3}{2}$ and $\frac{11}{4}$ using a common denominator of 4. Add to determine how much paint Caleb started with. He started with $\frac{17}{4}$ cans of paint.

$$\frac{17}{4} - \frac{19}{5} = \frac{85}{20} - \frac{76}{20}$$
$$= \frac{9}{20}$$

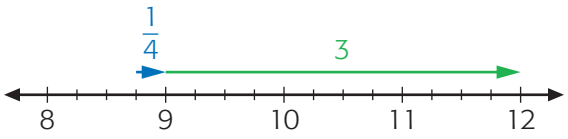
Rename $\frac{17}{4}$ and $\frac{19}{5}$ using a common denominator of 20. Subtract to determine how much paint is left.

There is $\frac{9}{20}$ of a can left.

Example 4 | Subtracting on a number line

Lang just turned 12. His sister is $8\frac{3}{4}$. How much older is he?

Solution



$12 - 8\frac{3}{4} = 3\frac{1}{4}$

It is $\frac{1}{4}$ from $8\frac{3}{4}$ to 9.

It is 3 from 9 to 12.

It is $3\frac{1}{4}$ from $8\frac{3}{4}$ to 12.

Lang is $3\frac{1}{4}$ years older than his sister.

A Checking

1. Calculate.

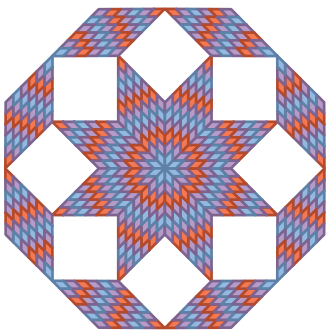
a) $5\frac{1}{4} + \frac{3}{8}$ **b)** $5\frac{2}{3} - 3\frac{1}{4}$ **c)** $3 - 1\frac{3}{4}$ **d)** $5\frac{7}{8} + 2\frac{5}{6}$

2. Jane is helping Oshana make her Bear Claw quilt. Suppose that Jane had 18 blue squares and made 2 quilt sections. How many blue squares would she have left?

B Practising

3. Calculate.

a) $3\frac{1}{4} + \frac{2}{3}$ **b)** $5\frac{1}{3} + \frac{3}{5}$ **c)** $4 - 2\frac{1}{5}$ **d)** $2\frac{2}{3} - \frac{3}{4}$



4. Calculate.

a) $3\frac{2}{3} - 1\frac{1}{3}$ **b)** $4\frac{2}{5} - 2\frac{4}{5}$ **c)** $3\frac{2}{5} + 5\frac{1}{6}$ **d)** $6\frac{3}{5} + 2\frac{3}{4}$

5. Ethel had $10\frac{1}{2}$ white squares before she made this star blanket. How many white squares does she have now?

6. Jasleen goes to bed 3 h after dinner. Yesterday, after dinner, she spent $1\frac{1}{2}$ h on her homework and $\frac{2}{3}$ h on the phone. How much time did she have left before bedtime? How do you know that your answer is reasonable?

7. Derrick's class wants to fill 2 boxes with school supplies for an orphanage in Ukraine. They have filled $1\frac{3}{4}$ of the boxes. How much more do they need to fill?

8. Use estimates to order these differences from least to greatest.

A. $5\frac{1}{3} - 4\frac{1}{2}$

B. $6\frac{1}{2} - 3\frac{2}{9}$

C. $12\frac{3}{8} - 9\frac{4}{5}$

D. $7\frac{2}{3} - 2\frac{6}{8}$

9. This week, Anita practised piano for $3\frac{1}{2}$ h, played soccer for $6\frac{1}{4}$ h, and talked on the phone for $4\frac{1}{3}$ h.

a) How many hours did Anita spend practising piano and playing soccer?

b) How many more hours did Anita spend playing soccer than talking on the phone?

10. Tori plays the tuba in a band. For a song that is 36 measures long, she plays for $4\frac{1}{2}$ measures, rests for $8\frac{3}{8}$ measures, plays for another 16 measures, rests for $2\frac{1}{4}$ measures, and plays for the last section. How many measures are in the last section?

11. Aviv cut out the ads on 5 pages of a newspaper. When he put the ads together, they filled $1\frac{1}{3}$ pages. Then he put the rest of the pages together. How many pages did they fill?

12. Describe a situation in which you might calculate $3\frac{1}{4} - 1\frac{1}{2}$.

13. When can the sum of two mixed numbers be a whole number? Explain.

14. Kevin added $4\frac{\square}{\square} + 3\frac{\square}{\square}$. What could the whole number part of the answer be? Why?

15. To calculate $7\frac{1}{8} - 2\frac{2}{3}$, Lee added $\frac{1}{3}$ to $4\frac{1}{8}$. Why do you think Lee did this?

16. Explain the reasoning for each statement.

a) It is easier to estimate $12\frac{1}{5} - 2\frac{1}{3}$ as mixed numbers than as improper fractions.

b) To calculate $2\frac{1}{2} + 3\frac{2}{3}$, add 4 to $2\frac{1}{2}$ and then subtract $\frac{1}{3}$.

17. Describe three strategies you can use to calculate $4\frac{1}{2} - 2\frac{5}{6}$.

2.11 Communicate about Estimation Strategies

GOAL

Explain how to estimate sums and differences of fractions and mixed numbers.

LEARN ABOUT the Math

Megan ordered 7 pizzas for a math class party. The students ate all but $2\frac{1}{3}$ pizzas. Megan says, "We ate almost 5 pizzas." Jacob says, "How do you know?"

Megan explains, "When I estimate with fractions, I like to use whole numbers. We started with 7 pizzas. There are about 2 left, and $7 - 2 = 5$."

How can Megan improve her explanation?

Megan can show more detail in her explanation.

I drew 7 circles to represent the pizzas.

I coloured 2 pizzas to show that they are left.

I coloured $\frac{1}{3}$ of the next pizza to show that it is also left. Almost 5 pizzas are gone, so we ate more than $4\frac{1}{2}$ pizzas, but less than 5. I think a good estimate is all we need.

Communication *Checklist*

- ✔ Did you show all the necessary steps?
- ✔ Were your steps clear?
- ✔ Did you include words to describe your model, as well as pictures?
- ✔ Did your words support your use of the models?

A. Use the Communication Checklist to explain how Megan improved her explanation.

B. Edit Megan's explanation. Explain how your changes improve it.

Reflecting

C. Why was it reasonable for Megan to estimate, rather than calculate? Explain.

D. Why does a visual model help to explain an estimation strategy?

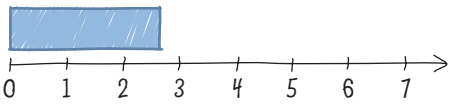
WORK WITH the Math

Example | Estimating a total

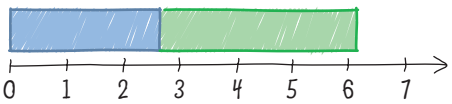
Ryan is building a birdhouse. He needs $2\frac{2}{3}$ boards for one part of the birdhouse and $3\frac{1}{2}$ boards for another part. About how many boards does Ryan have to buy? Why does he just need an estimate?

Ryan's Solution

I just need an estimate because I have to buy either 6 or 7 boards. I can't buy part of a board.



$2\frac{2}{3}$ is a little more than $2\frac{1}{2}$.



If I added $2\frac{1}{2}$ and $3\frac{1}{2}$, I'd get 5 wholes and 2 halves. That's 6 whole boards. The total is a little more than 6 boards, since $2\frac{2}{3}$ is a little more than $2\frac{1}{2}$. I have to buy 7 boards.

A Checking

1. Mia has $4\frac{1}{4}$ packages of modelling clay. She wants to estimate how many packages of clay will be left if her brother uses $2\frac{1}{2}$ packages. Here is the beginning of her explanation to her brother: "$4\frac{1}{4}$ is a little more than 4. The distance from $2\frac{1}{2}$ to 3 is $\frac{1}{2}$." Complete her explanation. Use the Communication Checklist.

B Practising

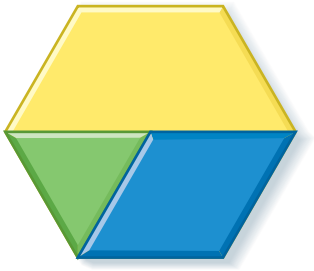
2. George's family had $5\frac{1}{2}$ packages of noodles. One Sunday, George used $1\frac{5}{6}$ packages to make a casserole. About how many packages are left? Why is an estimate all he needs?

3. Karen's grandmother has 10 scoops of flour. One batch of bannock uses $2\frac{1}{3}$ scoops of flour. About how many batches can she bake? Why is an estimate all she needs?

4. Braydon and Winnie are each building a bridge with straws for a science project. They have 9 bags of straws. Braydon thinks that he will use $3\frac{4}{5}$ bags of straws. Winnie thinks that she will use $2\frac{3}{4}$ bags of straws. About how many bags of straws will be left? Explain.

5. Suki, Lee, and Janice have collected the same number of pencils for the orphanage in Ukraine. When they put their pencils together, they have almost 5 full boxes of pencils. About how many boxes of pencils does each person have? Explain.

Chapter Self-Test



1. Order $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{8}{5}$ from least to greatest.

2. Rename in lowest terms.
   a) $\frac{8}{24}$ **b)** $\frac{21}{35}$ **c)** $\frac{36}{15}$

3. Two fractions can be renamed as 18ths. What could their denominators be?

4. **a)** Which part of the pattern block model shows $\frac{1}{3} + \frac{1}{6}$?
   b) Calculate $\frac{1}{3} + \frac{1}{6}$.

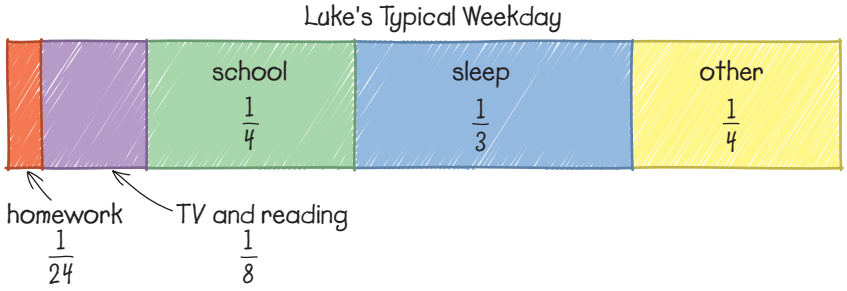
5. Calculate. Then estimate to show that your answers are reasonable.
   a) $\frac{3}{8} + \frac{1}{4}$ **b)** $\frac{2}{5} + \frac{3}{4}$

6. Francis wrote a story on his computer for $\frac{1}{2}$ of an hour and then played computer games for $\frac{1}{4}$ of an hour. Write an equation to describe the fraction of an hour that Francis used his computer.

7. Calculate. Show your work.
   a) $\frac{3}{8} - \frac{1}{4}$ **b)** $\frac{4}{5} - \frac{3}{4}$

8. Heather is earning money to buy a new stereo. She has earned $\frac{4}{5}$ of the amount she needs. What fraction of the amount does she still need to earn?

9. Luke drew a picture to show how he spends a typical weekday.



a) What fraction of the day does Luke not spend sleeping or in school?

b) How much more of the day does Luke spend in school than on homework?

c) Use the picture to make up a problem that has $\frac{7}{12}$ as the answer.

10. Calculate $\frac{3}{8} + \frac{2}{5}$. Explain what you did.

11. Calculate. Show your work.

a) $\frac{5}{6} - \frac{4}{9}$

b) $\frac{7}{10} - \frac{3}{8}$

12. Calculate. Show your work.

a) $2\frac{3}{4} + 3\frac{8}{9}$

b) $4 - \frac{1}{10}$

c) $3 - 2\frac{1}{5}$

d) $4 - 2\frac{3}{7}$

13. The difference between two mixed numbers is a whole number. What do you know about the two mixed numbers?

What Do You Think Now?

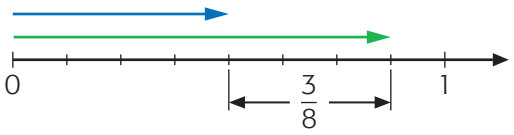
Revisit What Do You Think? on page 43. How have your answers and explanations changed?

Chapter Review

Frequently Asked Questions

Q: **What model is helpful for adding and subtracting fractions?**

A: You can use a number line with different numbers of sections. For example, the following number line was used to subtract $\frac{7}{8} - \frac{1}{2}$:



$$\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$$

Q: **How do you add or subtract fractions using equivalent fractions?**

A: Choose a common multiple of the two denominators. Then write a new equation, using equivalent fractions that have the common multiple as their denominators. For example, to add $\frac{3}{4} + \frac{3}{5}$, use the common denominator 20, since 20 is a common multiple of 4 and 5.

$$\frac{3}{4} \overset{\times 5}{=} \frac{15}{20} \quad \frac{3}{5} \overset{\times 4}{=} \frac{12}{20}$$

$$\frac{3}{4} + \frac{3}{5} = \frac{15}{20} + \frac{12}{20} = \frac{27}{20} \text{ or } 1\frac{7}{20}$$

Q: **How do you add mixed numbers, such as $2\frac{3}{4} + 4\frac{1}{2}$?**

A1: Add the whole numbers and the fractions separately.

- Add the whole numbers: $2 + 4 = 6$
- Add the fractions: $\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4}$, and $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ or $1\frac{1}{4}$.
- Then add the whole number sum and fraction sum.

  $6 + 1\frac{1}{4} = 7\frac{1}{4}$

A2: Rename both mixed numbers as improper fractions. Rename the improper fractions with common denominators if you need to, and then add.

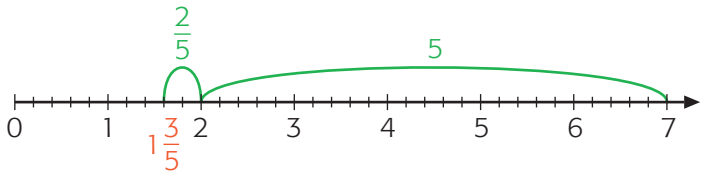
$2\frac{3}{4} = \frac{11}{4}$ and $4\frac{1}{2} = \frac{9}{2}$

$\frac{11}{4} + \frac{9}{2} = \frac{11}{4} + \frac{18}{4}$

$= \frac{29}{4}$ or $7\frac{1}{4}$

Q: How do you subtract a mixed number from a whole number?

A: Use a number line to determine the difference between the numbers. For example, to subtract $7 - 1\frac{3}{5}$, draw jumps that are easy to add from $1\frac{3}{5}$ to 7.



The difference between $1\frac{3}{5}$ and 2 is $\frac{2}{5}$. The difference between 2 and 7 is 5. The total difference is $\frac{2}{5} + 5$, or $5\frac{2}{5}$. So, $7 - 1\frac{3}{5} = 5\frac{2}{5}$.

Q: How do you subtract mixed numbers, such as $7\frac{1}{3} - 2\frac{3}{4}$?

A1: Subtract the fractions and whole numbers separately. Rename the fractions using fractions with a common denominator. If the fraction being subtracted is greater than the original fraction, regroup one whole. For example, you can subtract $7\frac{1}{3} - 2\frac{3}{4}$ by regrouping:

Since $\frac{1}{3} < \frac{3}{4}$, regroup. $7\frac{1}{3} = 6\frac{4}{3}$ $\qquad 2\frac{3}{4} = 2\frac{9}{12}$

$= 6\frac{16}{12}$

Subtract. $\qquad 6\frac{16}{12} - 2\frac{9}{12} = 4\frac{7}{12}$

A2: Rename both mixed numbers as improper fractions. Rename the improper fractions with common denominators if you need to, and then subtract.

$$7\frac{1}{3} = \frac{22}{3} \text{ and } 2\frac{3}{4} = \frac{11}{4}$$

$$\frac{22}{3} - \frac{11}{4} = \frac{88}{12} - \frac{33}{12}$$

$$= \frac{55}{12}$$

$$= \frac{48}{12} + \frac{7}{12}$$

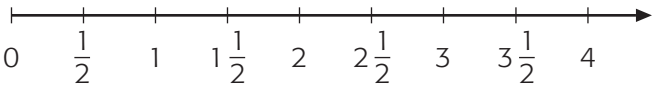
$$= 4\frac{7}{12}$$

Practice

Lesson 2.1

1. Name three fractions between $\frac{1}{2}$ and $\frac{5}{4}$.

2. a) Place $\frac{8}{7}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{15}{4}$ on the number line.



b) List them in order from least to greatest.

3. Rename in lowest terms.

a) $\frac{6}{10}$ **b)** $\frac{12}{36}$ **c)** $\frac{20}{12}$ **d)** $\frac{81}{36}$

Lesson 2.3

4. Calculate.

a) $\frac{6}{12} + \frac{7}{12}$ **b)** $\frac{5}{8} - \frac{1}{4}$ **c)** $\frac{5}{8} + \frac{1}{4}$ **d)** $\frac{3}{5} + \frac{1}{2}$

Lesson 2.4

5. Calculate.

a) $\frac{4}{7} - \frac{1}{3}$ **b)** $\frac{11}{12} - \frac{2}{3}$ **c)** $\frac{5}{6} - \frac{2}{3}$ **d)** $\frac{3}{4} - \frac{3}{5}$

6. a) Marian has $\frac{2}{3}$ of a bag of bagels. She finds another $\frac{1}{4}$ bag of bagels and puts these bagels in the first bag. What fraction of the first bag is now full of bagels?

b) Marian has a third bag of bagels that is $\frac{5}{6}$ full. What fraction describes how many more bagels are in the first two bags combined than in the third bag?

Lesson 2.6

7. Nunavut covers about $\frac{1}{5}$ of Canada's area. Manitoba covers about $\frac{1}{15}$ of Canada's area. What fraction describes how much more of Canada is covered by Nunavut than by Manitoba?

Lesson 2.7

8. Estimate whether each sum is greater than 1. Explain your estimate.

a) $\frac{2}{3} + \frac{5}{7}$ **b)** $\frac{5}{6} + \frac{1}{7}$

9. Calculate each sum in question 8. Use your estimates to verify your calculations.

Lesson 2.9

10. Calculate using equivalent fractions. Show your work.

a) $\frac{3}{5} + \frac{2}{7}$ **b)** $\frac{8}{9} + \frac{2}{3}$ **c)** $\frac{7}{10} - \frac{2}{3}$ **d)** $\frac{2}{3} - \frac{3}{5}$

Lesson 2.10

11. Calculate. Show your work.

a) $\frac{3}{10} + 2\frac{3}{5}$ **b)** $4\frac{5}{9} + \frac{2}{3}$ **c)** $6 - 2\frac{2}{7}$

12. Kyle has 3 h to complete a technology project. He thinks that he will need $1\frac{5}{6}$ h to design and build. How much time will he have to write the report for his project?

Lesson 2.11

13. Estimate whether each sum is between 1 and 3. Explain your estimate.

a) $1\frac{1}{2} + 1\frac{1}{4}$ **b)** $\frac{5}{6} + \frac{1}{10}$ **c)** $1\frac{2}{3} + 2\frac{4}{7}$ **d)** $\frac{3}{5} + 1\frac{9}{13}$

Chapter Task

Task | Checklist

- ✔ Did you explain each step of each calculation?
- ✔ Did you present your combinations clearly?
- ✔ Did your solutions answer the questions?

New Car Lot

Your family has opened a car lot. You need to decide which models and colours of vehicles to buy. You have surveyed visitors to the car lot. Unfortunately, you spilled water on your results, so two fractions are missing.

Preferred model	four-door family car	jeep	truck	sports car
Fraction	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	

Preferred colour	silver	black	red	green	blue	beige
Fraction	$\frac{1}{4}$	$\frac{1}{10}$		$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$

What fraction of each model/colour combination should you order?

A. What fraction of visitors prefer sports cars? Explain.

B. What fraction of visitors prefer red vehicles? Show your work.

C. You ordered 12 cars. Use fractions to describe the models and the colours of the cars. Explain why you ordered these 12 cars.

D. Use addition and subtraction to make your own fraction problem about the car lot. Solve your problem.