
1
2
3
5
6
7
8
9
10

Chapter 10

Probability

GOAL

You will be able to

- conduct probability experiments
- describe probabilities using ratios, fractions, and percents
- determine all the possible outcomes for a probability experiment using a tree diagram, an organized list, or a table
- compare theoretical and experimental probabilities

Suppose that you are conducting probability experiments. What is the probability of getting a 6 in each experiment?

Chapter 10

Getting Started

YOU WILL NEED
- 2 six-sided dice
- counters

Lucky Seven

Matthew and Fiona are playing Lucky Seven.

Is Lucky Seven a fair game?

Lucky Seven Rules

1. Play with a partner.
2. Place eight counters in a pile between you.
3. Each of you rolls a die.
4. If the sum of the two dice is 5, 6, 7, or 8, Player 1 gets a counter. If the sum is any other number, Player 2 gets a counter.
5. The winner is the player with the most counters after eight rounds.

Communication | *Tip*

In this chapter, assume that a die is a cube numbered 1 to 6, unless it is described differently.

A. Play Lucky Seven with a partner. Decide who is Player 1 and who is Player 2. Play two rounds. Record the results.

Roll sum	Player 1	Player 2
7	✓	
10		✓

The ✓ shows which player won a counter.

B. Predict which is true:
- Player 1 is more likely to win.
- Player 2 is more likely to win.
- The two players are equally likely to win.

C. Roll another six times. Record who wins.

D. You have now rolled eight times. Record the **probability** of each player winning a counter as a ratio in the form number of counters won : number of rolls.

E. Combine your results with the results of two other pairs. Use the combined data to write a ratio for the probability of Player 1 winning a counter.

F. Use the combined data to write a ratio for the probability of Player 2 winning a counter.

G. How accurate was your prediction in part B? Is Lucky Seven a fair game? Explain why or why not.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. The probability of getting a tail when tossing a fair coin is the same as the probability of rolling an even number with a fair die.
2. The probability of rolling two 1s on two dice is twice as great as the probability of rolling one 1 on one die.
3. The experimental probability of a result will always be close to the theoretical probability of the result.

10.1 Exploring Probability

YOU WILL NEED

- 2 six-sided dice

GOAL

Determine the experimental probability of an event.

event

a set of one or more outcomes in a probability experiment; for example, the event of rolling an even number with a six-sided die consists of the outcomes of rolling a 2, a 4, and a 6

EXPLORE the Math

Julie and Nolan are playing a game.

- They each roll two dice.
- Julie adds her numbers.
- Nolan subtracts his numbers (lower from higher).
- They each roll lots of times.

They want to compare the ratios for the two **events.**

Is it more likely that Julie will roll a sum of 8 or that Nolan will roll a difference of 2?

10.2 Representing Probabilities as Fractions and Percents

YOU WILL NEED

- a spinner with 20 sections, a pencil, and a paper clip
- 3 coins

GOAL

Express probabilities using fractions, percents, and number lines.

LEARN ABOUT the Math

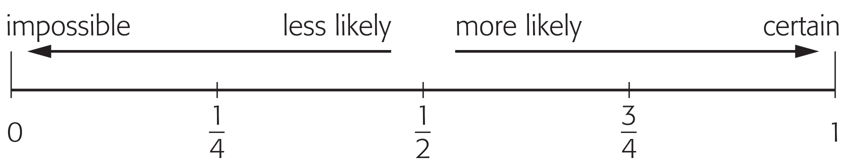
There are 20 students in a class: 13 boys and 7 girls. The teacher makes a spinner with all their names on it. The boys' names are in blue sections, and the girls' names are in green sections. Before the teacher calls on a student, she spins to select a name.

What is the probability of selecting a girl's name in 100 spins?

A. Spin a spinner like the one shown 10 times. After each spin, record whether the **favourable outcome** of selecting a girl's name occurred.

B. What fraction of the 10 spins represents the **experimental probability** of selecting a girl's name? Write this fraction as a percent.

C. Show the experimental probability of selecting a girl's name on the following probability line. How would you change this line to show the probability as a percent?



D. Explain why selecting a girl's name and selecting a boy's name are not **equally likely outcomes** when using the spinner.

favourable outcome
the desired result in a probability experiment

experimental probability
in a probability experiment, the ratio of the number of observed favourable outcomes to the number of trials, or repetitions, of the experiment

theoretical probability

the ratio of the number of favourable outcomes to the number of possible equally likely outcomes; for example, the theoretical probability of tossing a head on a coin is $\frac{1}{2}$, since there are 2 equally likely outcomes and only 1 is favourable

E. Write the **theoretical probability** of selecting a girl's name as a ratio, a fraction, and a percent.

F. Predict the number of times you might select a girl's name in 100 spins. Explain.

Reflecting

G. The probability line in part C begins with the label 0 and ends with the label 1. Why are these labels appropriate?

H. Did you use the fraction or the percent to predict the number of times a girl's name would be selected in 100 spins? How did you use it?

I. Why do the ratio, fraction, and percent all represent the same probability?

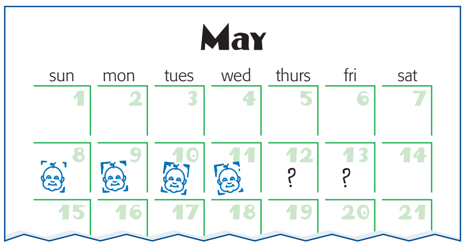
Communication | *Tip*

Instead of writing the words "probability of tossing a coin and getting heads," you can write *P*(heads); for example, *P*(heads) $= \frac{1}{2}$, or 0.5, or 50%.

WORK WITH the Math

Example | Representing an experimental probability

The first baby born in a hospital on each of the previous four days was a boy. Is the probability that a boy will be the first baby born on the next two days closer to $\frac{1}{2}$ or $\frac{1}{4}$?



Max's Solution

Heads and tails in a coin toss are equally likely outcomes. Boys and girls are about equally likely, too. I tossed coins to model the problem, and then estimated the probability.

Trial number	Both heads	Not both heads
1		✓
2	✓	
30		
Total	7	23

Since I had to think about two days, I tossed two coins and recorded the results. I made a head represent a boy and a tail represent a girl. If I tossed both heads, that meant a boy was born first on both days.

I tossed the two coins 30 times.

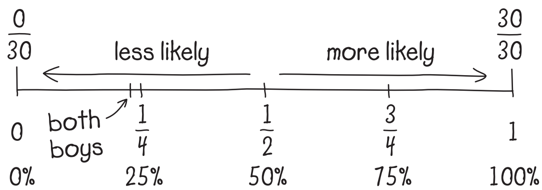
I calculated the probability that a boy would be born first on the next two days to be $P(2 \text{ boys}) = 7:30 = \frac{7}{30}$.

I wrote the experimental probability as a fraction. The numerator showed the number of favourable outcomes, and the denominator showed the total number of **trials**.

$7 \div 30 = 0.233333...$

I estimated this as 0.23, which is 23%.

It's easier to compare $\frac{7}{30}$ and $\frac{1}{4}$ as percents than as fractions.



I showed the probability on a probability line.

Instead of impossible, I wrote 0% and $\frac{0}{30}$.

Instead of certain, I wrote 100% and $\frac{30}{30}$.

23% is about halfway between 0 and $\frac{1}{2}$.

My probability line shows that the probability of the first baby born on the next two days being a boy is closer to $\frac{1}{4}$ than $\frac{1}{2}$. But I should probably combine my data with someone else's data since you should always repeat an experiment as many times as you can.

A Checking

1. Suppose that the class on the spinner had five more boys and five more girls. How would you calculate the probability of selecting a girl's name as a fraction? As a percent? As a ratio?

2. Roll a die 20 times. Record each roll and the experimental probability of each event as a ratio, a fraction, and a percent.
   a) rolling a 5
   b) rolling an even number
   c) not rolling a 4, 5, or 6
   d) rolling a 1

B Practising

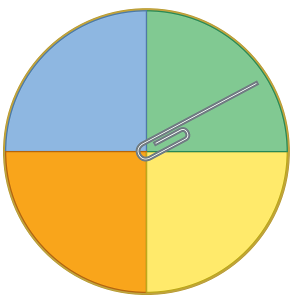
3. Joe read that the theoretical probability of winning a certain game is 30%.
   a) Write this probability as a ratio or a fraction.
   b) Show it on a probability line.

Reading Strategy

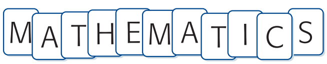
Ask three questions to help you understand this problem.
Share your questions with a partner.

4. Predict whether each theoretical probability below is closer to 10%, $\frac{1}{6}$, 50%, or 90%. Then test your prediction by rolling a die 20 times and reporting the experimental probability as a percent.
   a) P(roll is less than 6)
   b) P(roll is a 2)
   c) P(roll is more than 4)

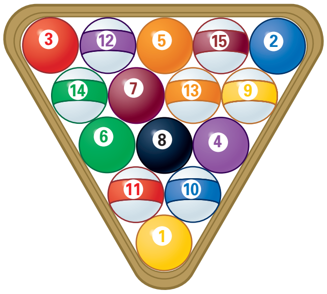
5. Show the theoretical probability of each event on a probability line.
   a) getting a tail when tossing a coin
   b) rolling a prime number with a six-sided die

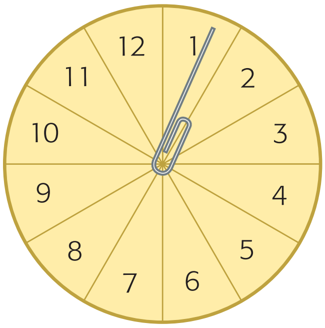


6. The four coloured areas on the spinner at the left are the same size. This means that the pointer is equally likely to land on all colours. Write each theoretical probability as a fraction and as a percent.
   a) P(yellow)
   b) P(green or yellow)
   c) P(black)
   d) P(not green)



7. Show each theoretical probability for the cards at the left on a probability line.
   a) P(vowel)
   b) P(consonant)
   c) P(letter with curved parts)





8. One ball is chosen at random from the rack of balls shown. Calculate each theoretical probability as a fraction or a ratio.

a) P(black)
b) P(10)
c) P(odd number)
d) P(even number)
e) P(solid purple, black, or orange)
f) P(number less than 20)

9. List the events in question 8 with a theoretical probability greater than 40%.

10. Use the spinner. Write the theoretical probability of each event to the nearest percent.

a) P(multiple of 3)
b) P(factor of 12)
c) P(prime number)
d) P(3, 5, or 8)
e) P(number less than 12)
f) P(multiple of 5)

11. Describe an event to match each probability.

a) 50% **b)** 25% **c)** $\frac{1}{31}$ **d)** $\frac{5}{6}$ **e)** 0 **f)** 1

12. A 10-sided die has sides numbered 1 to 10.

a) What fraction represents P(rolling an 8)?
b) What percent represents P(rolling an 8)?
c) How many 8s would you expect to roll in 100 rolls?
d) Did you use the fraction from part (a) or the percent from part (b) to answer part (c)?

13. Describe what each statement means.

a) The probability of the statement "I will win the race" is 0.
b) The probability of the statement "I will win the race" is 100%.
c) The probability of the statement "I will win the race" is $\frac{5}{10}$.

10.3 Probability of Independent Events

YOU WILL NEED
- 2 six-sided dice
- cards numbered 1 to 6

GOAL

Determine probability by identifying the sample space.

LEARN ABOUT the Math

Yan and Liam each roll a die. Yan wins if she rolls a 4. Liam wins if the sum of both their rolls is 4. Otherwise, it's a tie.

Is Yan more likely to win than Liam?

A. Yan created the following chart to show all the possible outcomes from rolling two dice. She uses W when she wins, L when she loses, and T when she ties. Complete the chart to show the experiment's **sample space.**

		Yan's roll					
		1	2	3	4	5	6
	1	T	T	L	W	T	T
	2				W		
Liam's roll	3				W		
	4				W		
	5				W		
	6				W		

I roll 4 and Liam rolls 2.

sample space

all the possible outcomes in a probability experiment

B. Calculate who is more likely to win the game.

C. Suppose that Liam and Yan change the game. Liam draws a card from a deck of five cards numbered 1 to 5, looks at the card, and puts it back in the deck. Then Yan draws a card. Yan still wins if she draws a 4, and Liam still wins if the sum of their draws is 4. List all the possible outcomes in this sample space.

D. Who is more likely to win the new game?

Reflecting

E. How could you have predicted that there would be 36 outcomes in the sample space in part A?

F. How do you know that Yan's chart in part A includes all the possible outcomes for the game?

G. In a third version of the game, Liam does not put his card back. Why were Yan's and Liam's card draws in part C **independent events,** but their draws in the new game not independent events?

independent events

two events are independent events if the probability of one is not affected by the probability of the other; for example, if you toss a coin and roll a die, the events are independent. The result of rolling the die does not depend on the result of tossing the coin.

WORK WITH the Math

Example | Using an organized list

Fiona's hockey team keeps a cooler full of apple juice, orange juice, cranberry juice, and water. The team mom pulls out bottles for players without looking. Fiona wondered what the probability is that the next two drinks that are pulled out will be the same.

Fiona's Solution

I used A for apple juice, C for cranberry juice, W for water, and O for orange juice. Then I listed all the combinations of two drinks possible. I listed them as first drink - second drink.

I used an organized list to see all the possible combinations of drinks that might occur.

A - A	C - A	W - A	O - A
A - C	C - C	W - C	O - C
A - W	C - W	W - W	O - W
A - O	C - O	W - O	O - O

If both drinks in an outcome were the same, I highlighted them in green. I counted the total number of outcomes and the number that I had highlighted in green.

P(2 of the same drink) = $\frac{4}{16}$

The probability is $\frac{4}{16}$ or $\frac{1}{4}$, which is 25%.

There were 16 outcomes, but only 4 had both drinks the same.

A Checking

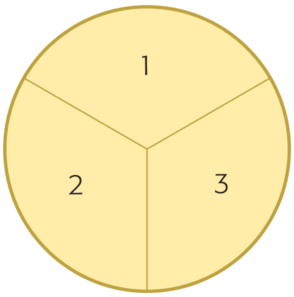
1. One experiment involves spinning a spinner and tossing a coin. Another experiment involves taking two coloured balls, one after the other, from a bag. Which experiment has outcomes that involve independent events? Explain your choice.

2. Suppose that you roll two dice. What is the theoretical probability of each event?
a) sum of 8
b) sum of 7
c) sum of 3

Second roll \ First roll	1	2	3	4	5	6
1	2	3				
2	3	4				
3	4					
4	5					
5	6					
6	7					

B Practising

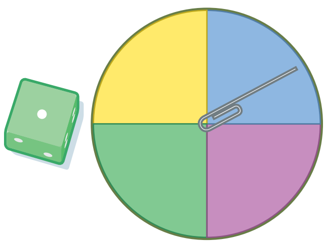
3. Suppose that you roll two dice. What is the theoretical probability of each event?
a) difference of 3 **b)** difference of 1 **c)** difference of 0



Second spin \ First spin	1	2	3
1			
2			
3			

4. Imagine spinning the spinner at the left twice.
a) Does the chart show all the possible outcomes? Explain.
b) How could you have predicted that there would be nine outcomes with two spins?
c) What is the theoretical probability of the sum of the two spins being an even number?
d) What is the theoretical probability of the sum of the two spins being greater than 2?

5. Deanna and Carol are playing a game. They roll a die twice and add the numbers they roll. A sum of 5 scores a point.
a) What is the probability of rolling a sum of 5?
b) What is the probability that Deanna will roll a sum greater than 5 on her next turn?
c) Why are the dice rolls independent events?



6. Suppose that you spin the spinner at the left and then roll the die. Determine the probability of each event.
a) P(yellow and 3)
b) P(yellow and anything except 3)
c) P(purple and even)

7. a) Describe a probability experiment with two independent events.
b) Describe a probability experiment with two events that are not independent.

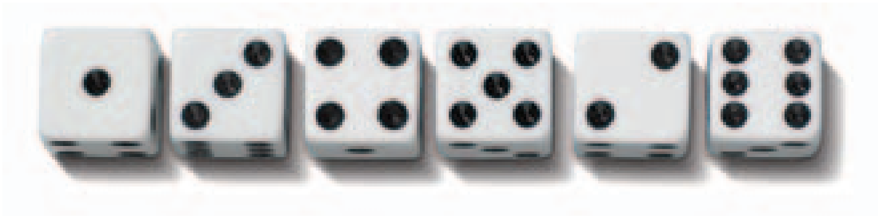
Chapter 10

Mid-Chapter **Review**

Frequently Asked Questions

Q: How can you describe a probability with numbers?

A: You can describe it as a ratio that compares the number of favourable outcomes to the number of possible outcomes. The ratio compares a part to a whole, so you can also write it as a fraction or a percent.



For example, the probability of rolling a multiple of 3 with a six-sided die can be written as $P(3 \text{ or } 6) = 2:6$. You could also write this as the fraction $\frac{2}{6}$, or $\frac{1}{3}$. Since $\frac{1}{3} = 0.3333 \ldots$, the probability is about 33%. You could show this on a probability line.

Q: Why must the value of a probability be between 0 and 1?

A: The least probability describes an event that never happens, or happens zero times. Therefore the least probability is

$$\frac{0}{\text{number of possible outcomes}} = 0.$$

The greatest probability describes an event that always happens. This means that the probability is 100%, or

$$\frac{\text{number of possible outcomes}}{\text{number of possible outcomes}} = 1.$$

Q: When are two events independent?

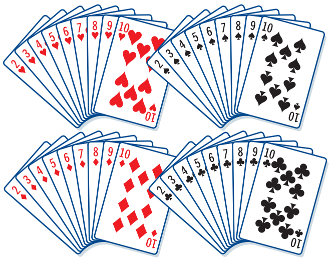
A: Two events are independent when the result of one event has no effect on the result of the other event.

For example, remove one ball from a bag, record its colour, put it back, and then repeat. The probability of removing the pink ball on the first draw is $\frac{1}{5}$. The probability of removing it on the second draw is still $\frac{1}{5}$. The two draws are independent events.

Now draw the pink ball first, but don't put it back. There is now no pink ball in the bag. The probability of drawing pink on the second draw is 0. These events are not independent.

Practice

Lesson 10.2



1. Fred shuffles the cards numbered 2 through 10 from a deck of cards. He then draws a card. Determine the theoretical probability of each event as a fraction and a percent, to the nearest whole number.

a) P(drawing a 10)
b) P(drawing a red card)
c) P(drawing a red 10)
d) P(drawing an odd card)
e) P(drawing a card less than 11)
f) P(drawing a king)

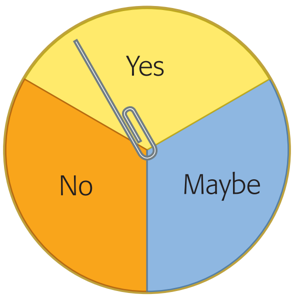
2. Katya rolled a regular six-sided die 16 times. Her experimental probabilities for three events are given:

$P(\text{even number}) = \frac{9}{16}$ $\quad P(\text{less than } 4) = \frac{8}{16}$ $\quad P(\text{exactly } 6) = \frac{1}{16}$

a) Which result matches the theoretical probability?
b) Which result is close to, but not identical to, the theoretical probability?
c) Which result is far from the theoretical probability?

3. A bag contains two red marbles, three green marbles, and seven black marbles. One marble is removed from the bag.

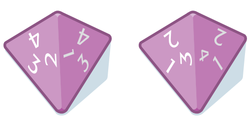
a) Write P(green) as a fraction and as a percent.
b) Predict the number of times a green marble will be selected if this experiment is carried out 50 times.



4. Raj is conducting an experiment. He spins the spinner at the left twice for each trial.

a) List all the possible outcomes for a trial.
b) Determine the probability of getting the same spin result in both spins of a trial.
c) Determine the probability of not getting a Yes in two spins.

Lesson 10.3



5. Two four-sided dice, each numbered 1 to 4, are rolled.

a) List all the possible outcomes in the sample space of this experiment.
b) Explain why the rolls of the dice are independent events.
c) Determine the probability that the sum of the dice will be 5.

MATH GAME

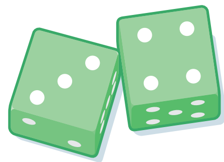
On a Roll

YOU WILL NEED

- two dice

The goal of this game is to be the first player to make 10 correct predictions.

Number of players: 2 to 4



How to Play

1. One player rolls two dice and calculates the product of the numbers rolled.
2. All the players predict whether the product of the next roll will be greater than, less than, or equal to this product.
3. The next player rolls the dice. Players who made a correct prediction score 1 point.
4. Players take turns rolling the dice and calculating the product. The game continues until a player has 10 points.

Roll	Result of roll	Product	My score	My prediction for next roll
1	3, 4	12		greater
2	3, 5	15	1	equal
3	1, 1	1	1 + 0 = 1	greater

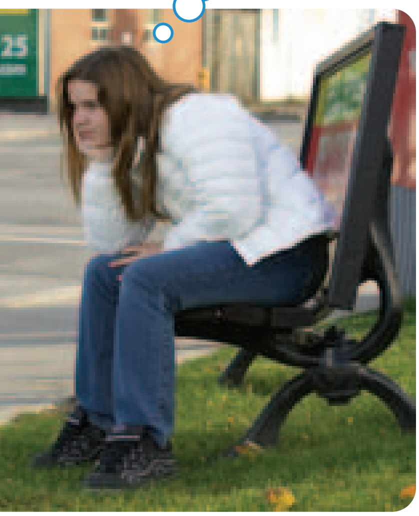
10.4 Solve Problems Using Organized Lists

YOU WILL NEED

- play money: loonies, dimes, and quarters

GOAL

Solve a problem using an organized list to identify the sample space.



LEARN ABOUT the Math

Julie is waiting for the school bus. She would like to buy a drink and a snack while waiting. She has five coins in her wallet. She knows that they must be quarters, dimes, or loonies, but she doesn't remember how many of each she has. For fun, she decides not to look, and to figure out the probability that she can buy only a snack or only a drink, but not both.

What is the probability that Julie will be able to buy only one item?

1 Understand the Problem

- I have five coins.
- Each coin is either a loonie, a dime, or a quarter.
- There may be more than one of some types of coins.
- There may be none of some types of coins.
- The snacks are \$2.00 each, and the drinks are \$2.00 each. This means that if the total value of the coins is at least \$2.00 but less than \$4.00, I can buy only one item.

2 Make a Plan

I'll write all the possible combinations of five coins in an organized list. This will help me see what combinations have a value of at least \$2.00 and less than \$4.00. I'll create my table using a pattern, so I won't miss or repeat any combinations.

❸ Carry Out the Plan

Loonies	Quarters	Dimes	Total value	Summary of possible outcomes	
5	0	0	$5.00	with 5 loonies,	
				1 combination	
4	1	0	$4.25	with 4 loonies,	
4	0	1	$4.10	2 combinations	
3	2	0	$3.50	with 3 loonies,	I highlighted the
3	1	1	$3.35	3 combinations	combinations with
3	0	2	$3.20		a total value of
2	3	0	$2.75	with 2 loonies,	at least $2.00
2	2	1	$2.60	4 combinations	and less than $4.00.
2	1	2	$2.45		There are 8 of
2	0	3	$2.30		these combinations
1	4	0	$2.00	with 1 loonie,	
1	3	1	$1.85	5 combinations	There are
1	2	2	$1.70		21 different
1	1	3	$1.55		combinations,
1	0	4	$1.40		and 8 of these
0	5	0	$1.25	with no loonies,	combinations
0	4	1	$1.10	6 combinations	allow me to buy
0	3	2	$0.95		only one item.
0	2	3	$0.80		The probability of
0	1	4	$0.65		being able to buy
0	0	5	$0.50		only one item is $\frac{8}{21}$.

❸ Look Back

I checked my table to make sure that I didn't miss any combinations. I counted to make sure that I had the same number of combinations with two quarters as I had with two loonies.

Reflecting

A. What pattern did Julie use?

B. How did using the pattern and organized list make it more likely that Julie listed all the possible combinations with no repeats?

WORK WITH *the Math*

Example | Solving a problem with an organized list

In a school checkers tournament, a win is worth 5 points, a tie is worth 2 points, and a loss is worth 1 point. If two players have the same number of points, then the player with more wins than ties is ranked higher. Yan has 10 points. What is the probability that she has more wins than ties?

Matthew's Solution

1 Understand the Problem

This is the information I know:

- Yan has 10 points.
- There could be some losses.
- Wins are worth 5 points, ties are worth 2 points, and losses are worth 1 point.

2 Make a Plan

I'll list all the combinations of wins, losses, and ties that add to 10 points. I'll start by looking at the number of possible wins. Then I'll think about ties and losses.

3 Carry Out the Plan

Combination for 10	Number of wins	Number of ties	Number of losses	
5 + 5	2	0	0	There are 10
5 + 2 + 2 + 1	1	2	1	ways to score
5 + 2 + 1 + 1 + 1	1	1	3	10 points.
5 + 1 + 1 + 1 + 1 + 1	1	0	5	
2 + 2 + 2 + 2 + 2	0	5	0	Only two ways
2 + 2+ 2 + 2 +1 + 1	0	4	2	have more wins
2 + 2 + 2 + 1 + 1 + 1 + 1	0	3	4	than ties.
2 + 2 + 1 + 1 + 1 + 1 + 1 + 1	0	2	6	
2 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1	0	1	8	
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1	0	0	10	

The probability of having 10 points and more wins than ties is $\frac{2}{10}$.

4 Look Back

There are patterns in the columns for wins, ties, and losses.
The patterns lead me to think that I have listed all the possible combinations.

A Checking

1. Suppose that you have two coins in your pocket. Each coin is either a penny, a nickel, a dime, or a quarter. The coins may have the same value.

a) How many different combinations are possible? The order in which you count the coins does not matter.

b) How many of these combinations sum to less than 20¢?

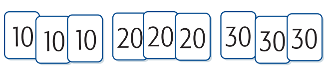
c) What is the probability that the two coins sum to less than 20¢?

B Practising

2. In a baseball tournament, teams get 5 points for a win, 3 points for a tie, and 1 point for a loss. Nathan's team has 29 points.

a) Show all the different combinations of wins, ties, and losses for Nathan's team using an organized list.

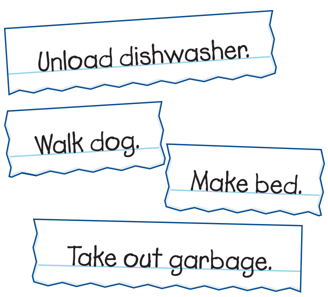
b) What is the probability that Nathan's team had more losses than ties?



3. Phil has the deck of nine cards at the left. Phil picks one card. He returns the card to the deck and picks another card. Then he multiplies the first card by the second card.

a) Show all of Phil's possible products.

b) Calculate the probability that Phil's product is over 500.



4. Each morning, John is the first to take one of these four jobs from the family daily job jar. What is the probability that he will get to walk the dog on both Monday and Tuesday?

5. Create a problem that can be solved using an organized list. Provide a complete solution.

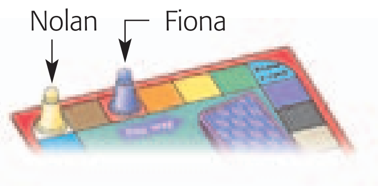
10.5 Using Tree Diagrams to Calculate Probability

YOU WILL NEED
- 2 six-sided dice
- counters
- grid paper

GOAL

Determine probabilities using a tree diagram.

LEARN ABOUT the Math



Nolan and Fiona are playing a board game with two counters and dice. Nolan is two spaces behind Fiona. They each roll a die and move forward the number of spaces rolled.

What is the probability that Nolan and Fiona will land on the same space?

tree diagram

a way to record and count all combinations of events, using lines to form branches to connect the two parts of the outcome; for example, the following tree diagram shows all the combinations that can happen if you toss a coin twice

1st toss	2nd toss	1st toss	2nd toss
H	H (HH)	T	H (TH)
	T (HT)		T (TT)

A. Nolan drew a **tree diagram** to show all the possible outcomes of rolling the two dice. Complete his diagram. How could you predict that there would be 36 branches?

Nolan's roll	Fiona's roll	Same space?	Nolan's roll	Fiona's roll	Same space?
	1 (orange)			1 (orange)	✓ (orange/orange)
	2 (yellow)			2 (yellow)	
1	3 (green)		3	3 (green)	
(brown)	4 (blue)		(orange)	4 (blue)	
	5 (purple)			5 (purple)	
	6 (grey)			6 (grey)	
	1 (orange)			1	
	2 (yellow)			2	
2	3 (green)		4	3	
(red)	4 (blue)		(yellow)	4	
	5 (purple)			5	
	6 (grey)			6	

The ✓ means that we landed on the same space.

B. Count the outcomes for which Nolan and Fiona land on the same space.

C. Calculate the theoretical probability of both students landing on the same space. Write it as a fraction.

Reflecting

D. Why does each branch represent one outcome?

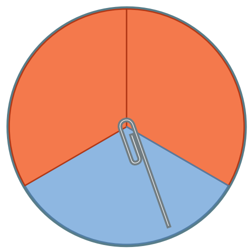
E. How did you use the tree diagram to determine the numerator and the denominator of the probability fraction?

F. How does using a tree diagram describe the sample space?

WORK WITH *the Math*

Example | Calculating using a tree diagram

Suppose that you spin the spinner twice. What is the probability of getting the blue section both times?



Max's Solution

First spin	Second spin	Outcome
1st red section (R1)	1st red section	(R1, R1)
	2nd red section	(R1, R2)
	blue section	(R1, B)
2nd red section (R2)	1st red section	(R2, R1)
	2nd red section	(R2, R2)
	blue section	(R2, B)
blue section (B)	1st red section	(B, R1)
	2nd red section	(B, R2)
	blue section	(B, B)

I assumed that each section has an equal chance of happening.

I listed all the possibilities in a tree diagram. I used R1 for one red section, R2 for the other red section, and B for the blue section.

The 9 branches represent 9 equally likely outcomes.

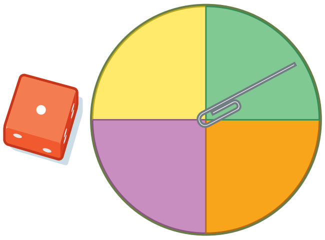
Only 1 of the 9 branches starts with the blue section and ends with the blue section, so 1 of the 9 outcomes is favourable.

P(blue section twice) = $\frac{1}{9}$

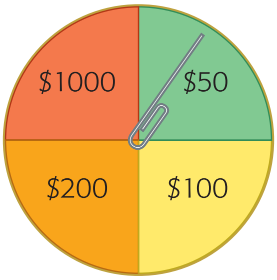
A Checking

1. Determine the theoretical probability that Nolan and Fiona will land on spaces next to each other. Use the tree diagram you made for the game Nolan and Fiona played.

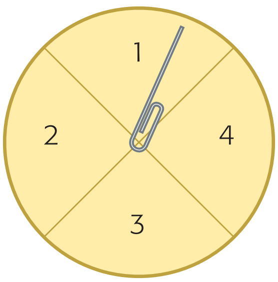
B Practising



2. Suppose that you roll the die and spin the spinner at the left.
   a) Show all the possible outcomes in the sample space using a tree diagram.
   b) Calculate *P*(anything but 5 and yellow).



3. Kaycee has won a contest. To determine the amount of her prize, she must spin this spinner twice. She will receive the sum of her two spins.
   a) Show all the possible outcomes by creating a tree diagram.
   b) What is the probability that Kaycee will receive more than the minimum amount but less than the maximum amount?
   c) What is the probability that Kaycee will receive more than $500?
   d) What is the probability that Kaycee will receive between $200 and $400?



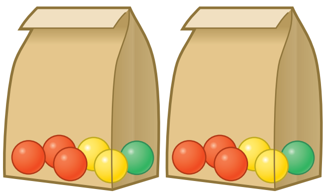
4 **a)** Suppose that the spinner is spun twice. List all the possible outcomes in a tree diagram.
   b) Determine the theoretical probability that the difference of the numbers will be 1.
   c) Determine the theoretical probability that the product of the numbers will be 4.
   d) Determine the theoretical probability that the second number will be 2 greater than the first number.

5. Omar has three T-shirts: one red, one green, and one yellow. He has two pairs of shorts: one red and one black. Answer the following questions using a tree diagram.

a) How many different outfits can Omar put together?

b) What is the probability that Omar's outfit will include a red T-shirt or red shorts?

c) Are choosing a T-shirt and shorts independent events? Explain.



6. Doug and Anna each have a bag containing three red balls, two yellow balls, and one green ball.
In Doug's experiment, he takes one ball from the bag, puts it back, and then takes another ball.
In Anna's experiment, she takes one ball from the bag, and then takes another ball without returning the first ball to the bag.

a) Draw a tree diagram for Doug's experiment.

b) Determine the probability that Doug will draw one red ball and one green ball using your tree diagram.

c) Why does Doug's experiment involve independent events, while Anna's experiment does not?

7. A probability experiment involves adding the numbers rolled on a four-sided die (with the numbers 1, 2, 3, and 4) and on a six-sided die.

a) Describe the sample space for this experiment using an organized list or a tree diagram.

b) Determine P(sum is less than 5).

8. How can you use multiplication to predict the number of branches on a tree diagram?

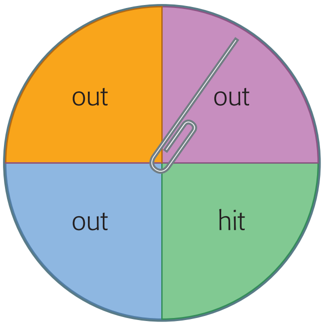
10.6 Comparing Theoretical and Experimental Probabilities

YOU WILL NEED

- a four-section spinner

GOAL

Compare theoretical and experimental probability for two independent events.



LEARN ABOUT the Math

Julie and Liam are playing a baseball board game in which a spinner determines whether a player gets a hit or strikes out.

? How does the theoretical probability that Julie will get two hits in her first two spins compare with the experimental probability?

Example 1 | Using a tree diagram

I listed all the possibilities in a tree diagram.

Yan's Solution

- hit
  - hit (hit, hit)
  - out (hit, out)
  - out (hit, out)
  - out (hit, out)
- out
  - hit (out, hit)
  - out (out, out)
  - out (out, out)
  - out (out, out)
- out
  - hit (out, hit)
  - out (out, out)
  - out (out, out)
  - out (out, out)
- out
  - hit (out, hit)
  - out (out, out)
  - out (out, out)
  - out (out, out)

I coloured each result so I could tell the section the spinner landed in.

There is only one outcome that means two hits.

Only 1 of the 16 outcomes has two hits.
The theoretical probability is

$P(H, H) = \frac{1}{16} = 0.0625$, or about 6%.

There are 16 outcomes, but only 1 with two hits.

Example 2 | Using an organized list

I did an experiment to determine the experimental probability.

Matthew's Solution

1st spin	2nd spin	1st spin	2nd spin
OUT	OUT	HIT	OUT
OUT	OUT	HIT	OUT
OUT	OUT	OUT	OUT
OUT	HIT	HIT	HIT
HIT	HIT	OUT	OUT
OUT	OUT	OUT	HIT
OUT	OUT	HIT	HIT
HIT	HIT	HIT	OUT
HIT	HIT	OUT	OUT
HIT	OUT	OUT	OUT

I carried out 20 trials, and I spun the spinner twice in each trial. Two hits in a row happened in 5 trials.

I suspect that there are too many HIT, HITs. I wonder if something is wrong with the spinner. I might check it. I'll also see what results my classmates got.

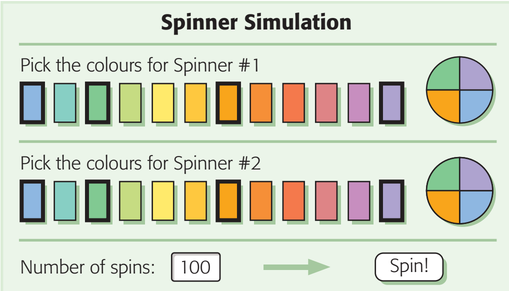
There were 5 results of HIT, HIT in 20.

$P(H, H) = \frac{5}{20} = 0.25$ or 25%.

Example 3 | Using a simulated spinner

I located a double spinner simulation.

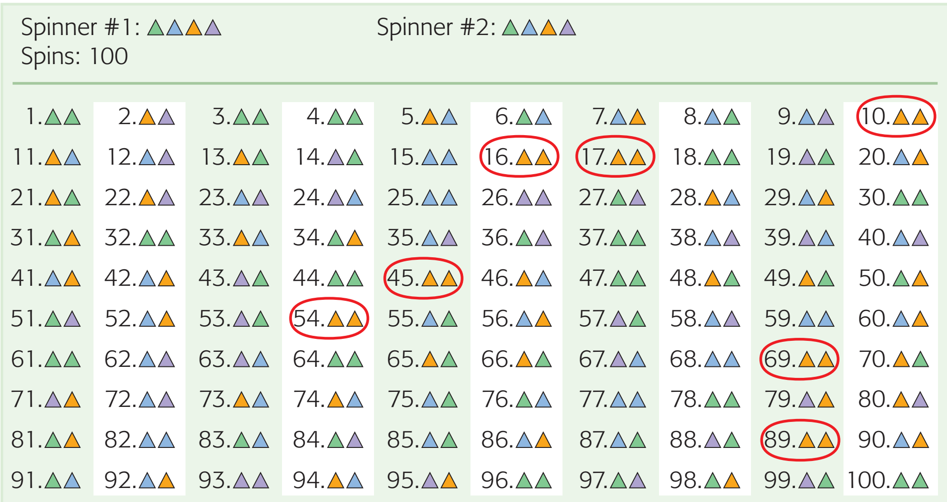
Fiona's Solution



I set the simulation to spin 100 times.

I used yellow to represent a hit. If two yellow sections came up in a spin, then I had two hits in a row.

Because the spinner lets me, I could spin 1000 times and get a better experimental result.



There were 7 results that represented two hits in a row in 100 trials. The experimental probability is $P(H, H) = \frac{7}{100} = 0.07$ or 7%.

My experimental probability and Yan's theoretical probability are very close.

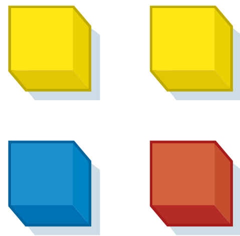
Reflecting

A. Why might the experimental probability be different from the theoretical probability?

B. Why do you think Fiona's data might be more reliable than Matthew's?

Example 4 | Comparing probabilities

A bag holds two yellow cubes, one blue cube, and one red cube. In an experiment you pick one cube from the bag, replace it, and then pick a second cube. Repeat this experiment 20 times. Compare the experimental and theoretical probabilities of drawing two cubes that are the same colour.



Solution

Y1
- Y1 (Y1, Y1)
- Y2 (Y1, Y2)
- R (Y1, R)
- B (Y1, B)

R
- Y1 (R, Y1)
- Y2 (R, Y2)
- R (R, R)
- B (R, B)

Y2
- Y1 (Y2, Y1)
- Y2 (Y2, Y2)
- R (Y2, R)
- B (Y2, B)

B
- Y1 (B, Y1)
- Y2 (B, Y2)
- R (B, R)
- B (B, B)

Draw a tree diagram to represent the different ways to remove the cubes. Use Y1 and Y2 for the yellow cubes, R for the red cube, and B for the blue cube.

There are 16 possible outcomes. Only 6 show cubes of the same colour.

P(2 cubes the same colour) $= \frac{6}{16}$

$= 0.375$, or about 38%

Write the theoretical probability as a percent so that it can be easily compared with the experimental probability.

In the spreadsheet experiment
- 0 and 1 represent yellow
- 2 represents red
- 3 represents blue

1st draw	0	3	3	3	0	2	1	2	1	3
2nd draw	0	0	1	3	3	3	1	3	3	1
1st draw	1	2	3	0	3	0	2	0	2	3
2nd draw	2	1	2	1	0	1	2	2	0	0

Make a spreadsheet for 20 draws. Use the random number generator. The favourable combinations will be
- 0-0, 0-1, 1-0, and 1-1 (both yellow)
- 2-2 (both red)
- 3-3 (both blue)

Run the experiment. Highlight the favourable combinations in the results.

There are 20 trials and 6 favourable outcomes.

P(2 cubes the same colour) $= \frac{6}{20} = 0.3$ or 30%

In a spreadsheet, you can repeat the experiment as many times as you like quickly and easily.

Reading Strategy

Summarize what you know about experimental and theoretical probability. Use a Venn diagram.

A Checking

1. A bag holds two red cubes, one blue cube, and one yellow cube. Darth is going to pick one cube from the bag, replace it, and then pick a second cube.

a) Calculate the theoretical probability that Darth will pick two different-coloured cubes.

b) Conduct the experiment 20 times. Compare the experimental probability with your answer in part (a).

c) Combine your results with those of your classmates. Compare the new experimental probability to your answer in part (a).

B Practising

2. a) What is the theoretical probability of rolling an even number on a die?

b) Roll a die 20 times. Combine your results with the results of four other students. Compare the experimental probability of rolling an even number with the theoretical probability.

3. a) Suppose that you roll two dice. What is the theoretical probability that both numbers will be a multiple of 3?

b) Conduct an experiment with at least 20 trials. What is your experimental probability for the event in part (a)?

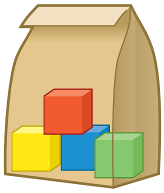
c) Why might the experimental probability be different from the theoretical probability?

d) Combine your results with the results of other students. What is the experimental probability now?

4. a) Suppose that you roll a die twice. What is the theoretical probability that you will roll a number greater than 3 before you roll a number less than 3?

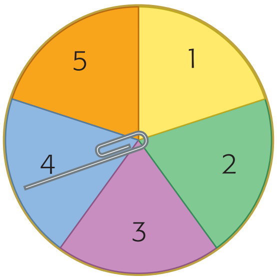
b) Compare the theoretical probability with an experimental probability. Use at least 20 trials.

c) How might you set up this experiment using computer technology?

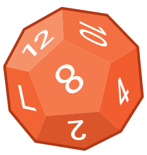


5. Anthony removes a block from the bag at the left, checks its colour, and puts it back. He then removes another block.
   a) What is the probability that Anthony will remove the red block on his second try?
   b) Why are the first and second block removals independent events?
   c) Do an experiment to determine an experimental probability for removing red on the second try. Combine your results with the results of other students to have more data. Compare your combined data with the theoretical probability.

6. How can you change the experiment in question 5 so that it does not involve independent events?



7. a) Determine the theoretical probability that when you spin this spinner twice, the product of the two spins will be an even number.
   b) Do an experiment to determine the experimental probability of the product being an even number.
   c) Compare your results as a class. What happened to the experimental probability as the number of trials increased?



8. Suppose that you roll a 12-sided die and toss a coin.
   a) Determine the theoretical probability of rolling an even number and tossing a head.
   b) Do an experiment to calculate the experimental probability of rolling an even number and tossing a head.
   c) Compare your answers in parts (a) and (b).

9. a) Mykola has calculated that the theoretical probability of a certain outcome in an experiment is 37%. He conducts the experiment five times and gets the following experimental probabilities for the outcome: 97%, 80%, 85%, 100%, and 92%. What advice would you give Mykola?
   b) Mae wants to determine the experimental probability that all three children in a family will be girls. She determined that the probability is 25%. What advice would you give Mae?

CURIOUS MATH

SIMPSON'S PARADOX

Suppose that you have two bags. Each bag contains some blue marbles and some black marbles. If you draw a blue marble from a bag, you win.

Here are two situations to investigate:

Situation A

Marbles	Bag 1	Bag 2
blue	5	3
black	6	4

Situation B

Marbles	Bag 1	Bag 2
blue	6	9
black	3	5

1. In both situations, you have a better probability of winning with bag 1. Explain why.
2. Suppose that you place the marbles from bag 1 of both situations into a single bag. Calculate the probability of winning using this bag.
3. Suppose that you place the marbles from bag 2 of both situations into a single bag. Calculate the probability of winning using this bag.
4. How is the probability of winning using combined bags different from the probability of winning using separate bags?

Chapter Self-Test

1. Write each probability as a fraction, a ratio, and a percent, based on Keith's rolls of a die.

Keith's Rolls of a Die

First 10 rolls	4	5	2	3	1	2	6	4	2	3
Next 10 rolls	2	1	1	3	1	6	5	4	6	4
Next 10 rolls	1	2	3	6	4	5	1	2	1	3

a) P(even number in the first 10 rolls)
b) P(odd number in all 30 rolls)
c) P(number less than 3 in the first 20 rolls)

2. Write each probability for Bridget's spins as a fraction.

Bridget's 25 Spins

2	3	3	1	4	1	2	5	1	2	2	3	1
1	2	4	5	2	4	1	1	2	3	3	1	

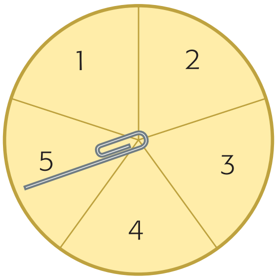
a) spinning a 2
b) spinning a 5
c) spinning a 1
d) spinning an even number

Roll	Result
1	4 and 2
2	5 and 5

3. Roll two dice 20 times. Record the result of each roll. Write each experimental probability as a percent.
a) two numbers greater than 3
b) two prime numbers
c) a sum of 3

4. You have cards numbered 1 to 5. In experiment A, you select a card, return it to the deck, and then select another card. In experiment B, you select one card and then select another card. Why does experiment A involve independent events, while experiment B does not?

5. Two identical dice are rolled at the same time. What is the theoretical probability of each event?
   a) 4 and 3
   b) 2 and another even number
   c) two consecutive numbers
   d) two numbers whose sum is a multiple of 3
   e) two numbers whose sum is a multiple of 5
   f) two numbers 3 apart



6. a) Draw a tree diagram to show all the possible outcomes when spinning this spinner twice.
   b) Determine the probability of spinning the same number twice. Use the tree diagram you drew in part (a).
   c) Determine the probability of spinning two numbers with a difference less than 3. Use your tree diagram.

7. A tree diagram has a total of 18 branches. It describes the possible outcomes when you toss a coin and spin a spinner. What might the spinner look like? Explain why.

8. Roll two dice 20 times. Record the product of the numbers you get for each roll.
   a) Write the experimental probability that the product will be even.
   b) Write the theoretical probability that the product will be even.
   c) Which probability is greater, experimental or theoretical?
   d) Suppose that you rolled 100 times, and your results did not seem to match the theoretical probability. What might be the reason?

9. In an experiment, a card is drawn from a deck of 10 cards numbered 1 through 10. Describe an event with each probability.
   a) 0 b) 1 c) $\frac{1}{2}$ d) 20% e) 4:10 f) 7:10

What Do You Think Now?

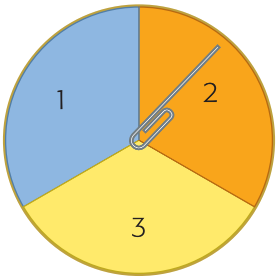
Revisit What Do You Think? on page 429. How have your answers and explanations changed?

Chapter Review

Frequently Asked Questions

Q: How can you solve a probability problem using an organized list or a tree diagram?

A: Organized lists and tree diagrams are strategies for listing all the outcomes in a sample space. For example, suppose that you want to know the probability of getting two spins that are the same with this spinner.

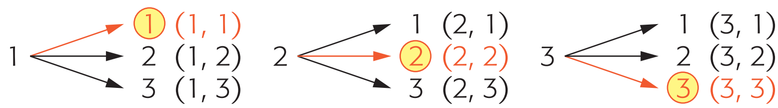


You can create an organized list to show the sample space and highlight the favourable outcomes.

1st spin is 1, 2nd spin is 1	1st spin is 2, 2nd spin is 1	1st spin is 3, 2nd spin is 1
1st spin is 1, 2nd spin is 2	1st spin is 2, 2nd spin is 2	1st spin is 3, 2nd spin is 2
1st spin is 1, 2nd spin is 3	1st spin is 2, 2nd spin is 3	1st spin is 3, 2nd spin is 3

$P(\text{both spins the same}) = \frac{3}{9} = \frac{1}{3}$

You can also use a tree diagram to represent all the outcomes, and then identify the favourable outcomes.



$P(\text{both spins the same}) = \frac{3}{9} = \frac{1}{3}$

Practice

Lesson 10.2

1. Suppose that you randomly choose an integer from 1 to 100. Write each probability as a fraction and a percent.

a) $P(\text{number is even})$

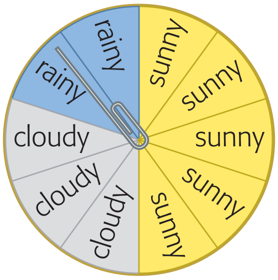
b) $P(\text{number has two digits})$

c) $P(\text{number is a multiple of 10})$

d) $P(\text{number is a multiple of 9})$

2. Supppose that you randomly choose another integer from 1 to 100. Describe an event with each probability.
   a) 25% **b)** 0 **c)** 100%

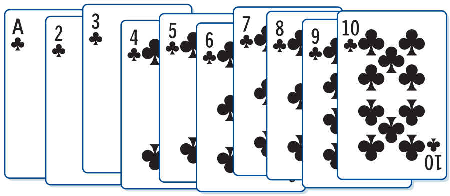
Lesson 10.3

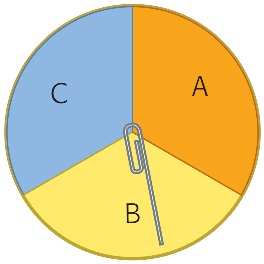


3. Imagine predicting the weather for the next two days using the weather spinner at the left.
   a) Explain why the spins are independent events.
   b) Explain why rainy, sunny, and cloudy are not equally likely events.

Lesson 10.4

4. Conduct an experiment in which you draw one card and spin the spinner once. List all the outcomes in the sample space. Then determine each theoretical probability using your list.
   a) *P*(greater than 7 and C) **b)** *P*(ace and 5)





Lesson 10.5

5. Consider an experiment in which you spin the spinner at the left once and toss a coin. Determine the sample space for the experiment by drawing a tree diagram. Then use your tree diagram to determine each probability.
   a) *P*(7 and H) **b)** *P*(odd and T)

Lesson 10.6

6. Roll one die twice.
   a) Determine the theoretical probability that the numbers rolled will be in increasing order.
   b) Determine the experimental probability of the outcome from part (a) by carrying out an experiment using 36 trials.
   c) Explain why the theoretical probability in part (a) might be different from the experimental probability in part (b).

Chapter 10

Chapter Task

Winning Races

Fiona, Julie, and Yan are the best runners in their school. They always come in first, second, or third in races. Each comes in first in 2 km races about $\frac{1}{3}$ of the time.

What is the probability that Julie will win both 2 km races in the June competitions?

A. Determine the theoretical probability that Julie will win both races using a tree diagram.

B. Explain why Julie's running experiment is a fair way to determine the experimental probability that Julie will win both races.

Julie's Running Experiment
Roll a die.
- If the result is 1 or 2, Fiona wins.
- If the result is 3 or 4, I win.
- If the result is 5 or 6, Yan wins.

Roll twice.
Repeat this experiment 25 times.

C. Determine the experimental probability that Julie will win both races using her experiment.

D. How do the probabilities in parts A and C compare? Which probability do you think is more accurate? Why?

E. Suppose that Julie trains really hard so she can win $\frac{1}{2}$ of the races. Each of the other two girls now wins only $\frac{1}{4}$ of the races. Would the tree diagram you drew in part A help you determine Julie's new probability of winning both races? Explain.

Task | Checklist

- ✔ Did you make sure that your tree diagram represented all the possible outcomes?
- ✔ Did you conduct the experiments an appropriate number of times?
- ✔ Did you explain your thinking?
- ✔ Did you support your conclusions?