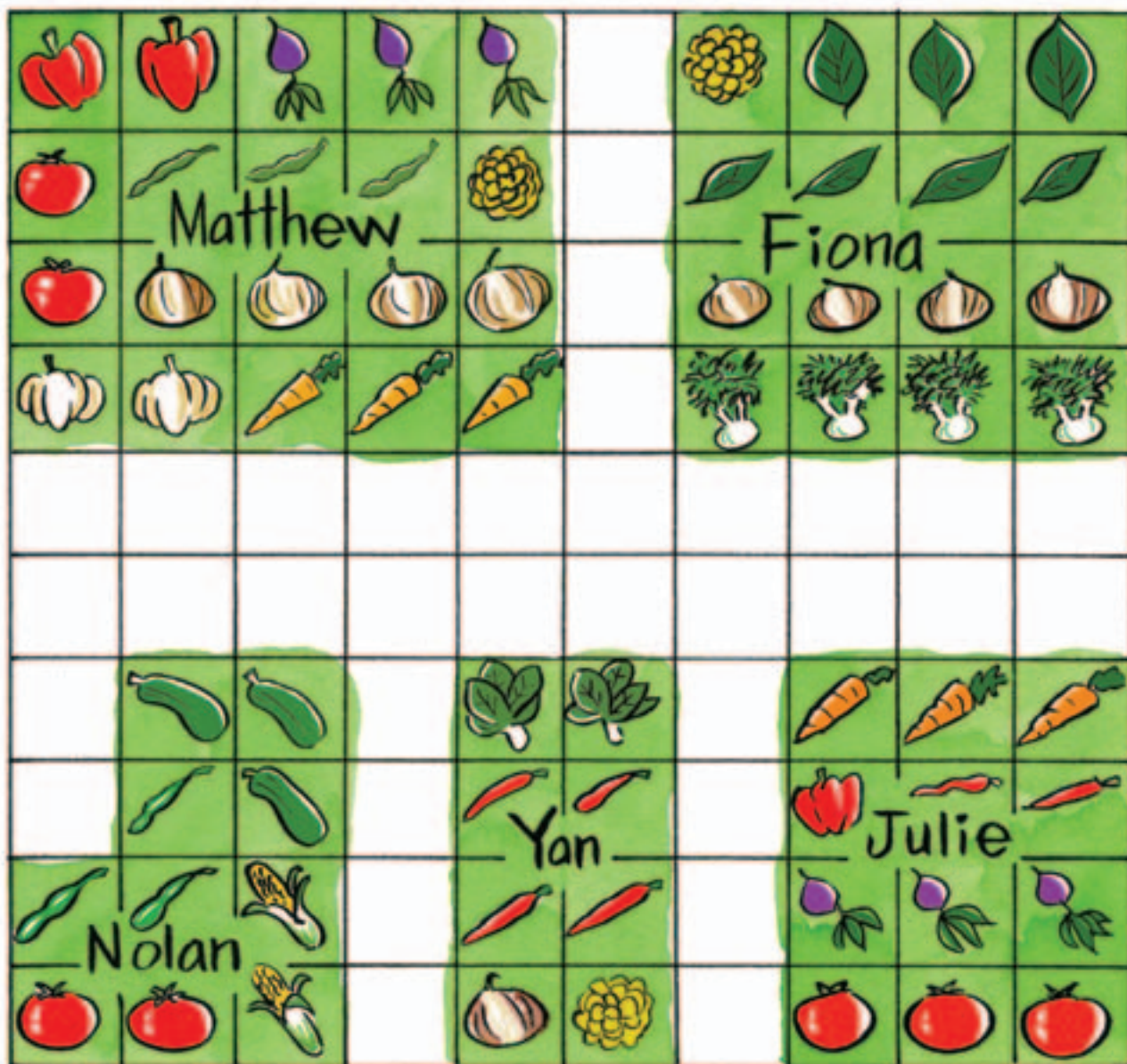# *Decimal Numbers*

GOAL

You will be able to

- add and subtract decimal numbers
- multiply and divide decimal numbers
- solve real-life problems that invlove decimal numbers
- express fractions as decimal numbers
- order decimal numbers

Decimals are found everywhere. What examples of decimals can you see in this party picture?

Getting **Started**

Comparing Numbers

Matthew's school has a garden where students can grow vegetables. In this plan of the garden, the white areas show the paths. Each small square is 1 m².

Decimal numbers can just be called decimals. In this book, they are called decimals.

What decimal describes the total area of the garden that is planted?

A. How many small squares are there in total?

B. What is the area of Matthew's patch?

C. Compare the area of Matthew's patch with the area of the whole garden. Write your answer as a decimal.

D. Compare the area of every other patch with the area of the whole garden. Write each answer as a decimal.

E. What part of the garden is planted? Write your answer as a decimal.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. Adding decimals is different from adding whole numbers.

2. Every decimal multiplication is related to a whole number multiplication.

3. $3.2 \div 0.4$ should be less than $3.2 \div 4$.

4. Every decimal is equivalent to more than one fraction.

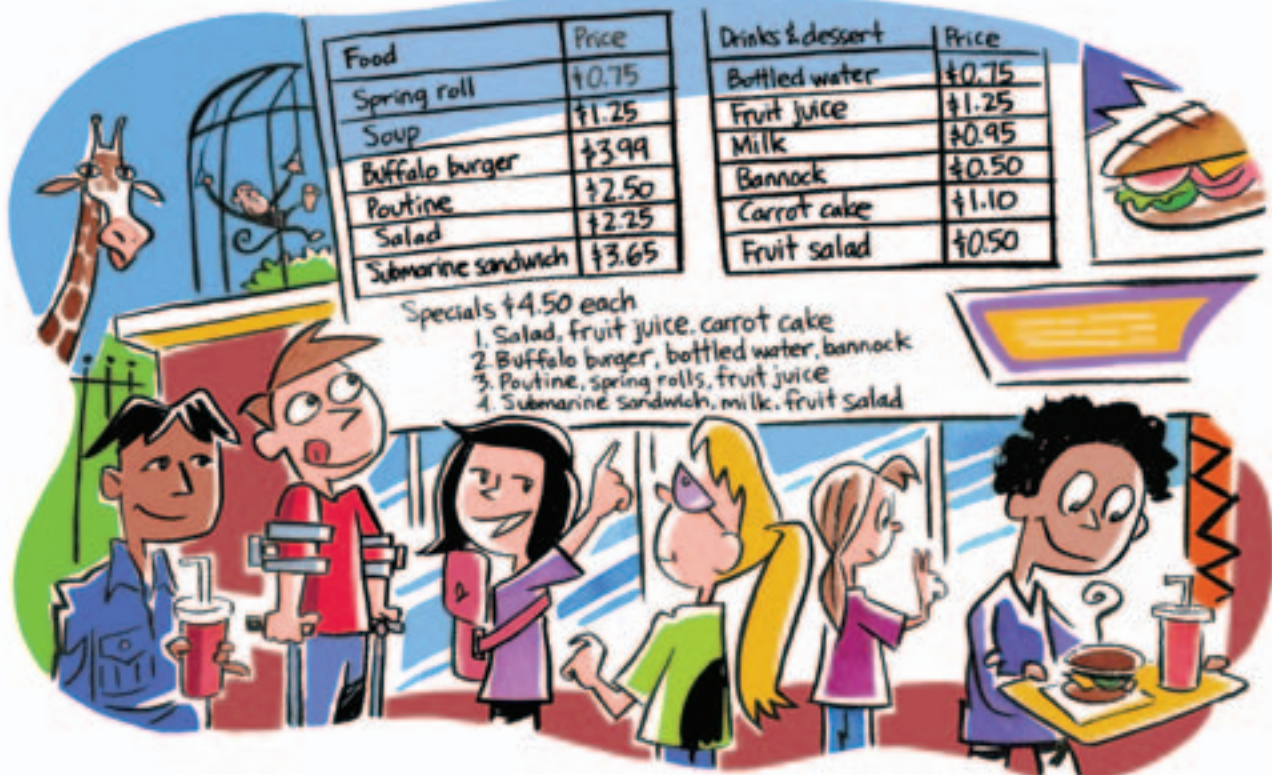# 3.1 Exploring Adding and Subtracting Decimals

GOAL

Add and subtract decimals using mental math.

EXPLORE the Math

You had $12.00 for a field trip to the zoo. You spent $6.50 for the admission fee. You can buy lunch with the money you have left.

? With the money you have left, is it better to buy a special or items separately from the menu?

3.2 Adding and Subtracting Decimals

YOU WILL NEED
- a place value mat
- base 10 blocks

GOAL

Develop strategies to add and subtract decimals.

LEARN ABOUT the Math

Matthew and Fiona are cutting a water pipe that is 15.000 m long into sections. Matthew needs sections that are 1.614 m, 5.341 m, and 2.172 m long. Fiona needs a section that is 5.000 m long.

Will there be enough pipe left for Fiona's section?

Example 1	**Estimating a sum and a difference**

I estimated the length of pipe I will need using front-end estimation.

Matthew's Solution

$\begin{array}{r} 1.614 \\ 5.341 \\ + 2.172 \\ \hline 8 \end{array}$	I added the ones.
$6 + 3 + 1 = 10 \text{ tenths}$ $= 1.0$	Then I added the tenths.
$8 + 1 = 9$	I added the two sums. I'll need about 9 m of pipe for my sections.
$15 - 9 = 6$	I subtracted my estimate from 15 m. I estimate that 6 m of pipe will be left. Since 6 m is greater than 5 m, there will be enough pipe left for Fiona's section.

Example 2 | Adding decimals

I calculated the length of pipe that Matthew will need.

Julie's Solution

Ones	Tenths	Hundredths	Thousandths

The three sections that Matthew will need are 1.614 m, 5.341 m, and 2.172 m. I modelled the lengths with base ten blocks on a place value mat.

I used the large block to represent 1 so that there would be a block to represent thousandths. The flat represents 0.1, the rod represents 0.01, and the small cube represents 0.001.

Ones	Tenths	Hundredths	Thousandths

I put blocks that were the same together, and I recorded my addition. There were 7 thousandths. I regrouped 10 of the 12 hundredths as 1 tenth.

$$\begin{array}{r} {}^{1} \\ 1.614 \\ 5.341 \\ +\ 2.172 \\ \hline 27 \end{array}$$

Ones	Tenths	Hundredths	Thousandths

I regrouped 10 of the 11 tenths as a one.

$$\begin{array}{r} {}^{1}\ {}^{1} \\ 1.614 \\ 5.341 \\ +\ 2.172 \\ \hline .127 \end{array}$$

Ones	Tenths	Hundredths	Thousandths

$$\begin{array}{r} {}^{1}\ {}^{1} \\ 1.614 \\ 5.341 \\ +\ 2.172 \\ \hline 9.127 \end{array}$$

Matthew will need 9.127 m of pipe for his sections.

There were 9 ones, 1 tenth, 2 hundredths, and 7 thousandths in total.

Example 3 | Subtracting decimals

To calculate the remaining length, I subtracted the amount that Matthew will need from the total length.

Fiona's Solution

15.000 – 9.127 is about 15 – 9 = 6.

I estimated first.

$$15\ 000 = 14\ 999 + 1$$
$$-9\ 127 \quad -9\ 127$$
$$5\ 872 + 1 = 5873$$

I thought of 15.000 as 15 000 thousandths and 9.127 as 9127 thousands and just calculated 15 000 − 9127.
I regrouped 15 000 to make the subtraction easier.
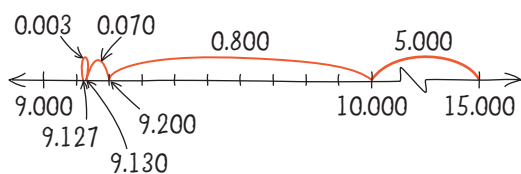
I'll have 5.873 m of pipe to work with.

I knew the answer had to be about 6, so I could easily place the decimal point.

Example 4 | Subtracting decimals using mental math

I imagined a number line, and calculated the length of pipe that will be left using mental math.

Liam's Solution

There will be 5.873 m of pipe left.

15.000 − 9.127 means "How far is it from 9.127 to 15.000?"

I calculated the difference in steps that made mental calculation easier.

Reflecting

A. How else could Julie and Fiona have regrouped to calculate their answers?

B. Why do you think Liam added 5, then 0.8, then 0.07, and then 0.003?

C. Which method would you have used for the subtraction? Why?

WORK WITH the Math

Example 5 | Adding and subtracting thousandths

For a science experiment, Amar and William need to add 0.800 g of salt to a beaker of water. Amar has measured 0.345 g of salt, and William has measured 0.406 g. How many more grams of salt do they need?

Solution

$$
\begin{array}{r}
0.345 \\
+\ 0.406 \\
\hline
0.751
\end{array}
$$

Add the amounts that Amar and William measured. Line up the digits to make sure that you add tenths to tenths, hundredths to hundredths, and thousandths to thousandths.

$$
\begin{array}{r}
\overset{7\,9\,10}{\cancel{0.800}} \\
-\ 0.751 \\
\hline
0.049
\end{array}
$$

Subtract the total amount that Amar and William have from the amount that they need. Line up the tenths digits, the hundredths digits, and the thousandths digits. Regroup so you can subtract.

The difference is 0.049, so Amar and William still need 0.049 g of salt.

A Checking

1. Estimate.
   a) $2.321 + 5.309 + 2.100$ **b)** $9.623 - 5.061$

2. Calculate.
   a) $3.05 + 4.26 + 0.63$ **b)** $4.563 - 2.937$

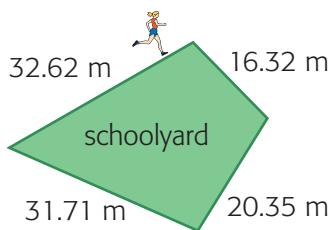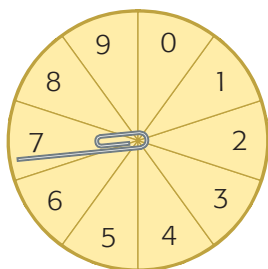### B Practising

3. Estimate.
   a) $2.5 + 12.6 + 20.9$ **c)** $78.615 - 29.321$
   b) $1.32 + 10.55 + 62.41$ **d)** $426.3 + 252.8 - 139.2$

4. Calculate.

a) $1.356 + 0.405 + 22.015$ **d)** $261.72 - 30.568$

b) $335.216 + 40.52 + 5.145$ **e)** $652.1 - 26.358$

c) $3.162 - 0.123$ **f)** $4.123 - 3.200$

5. Zoë is putting a fence around her garden. She needs 14.6 m of wire fence. She has three pieces already cut. These pieces are 6.6 m, 2.1 m, and 7.2 m long. Does she have enough? If so, how much will she have left over? If not, how much more does she need? Explain your reasoning.

6. Jocelyne and Martine ran 400 m. Jocelyne took 74.53 s, and Martine took 89.34 s. How many seconds faster was Jocelyne than Martine?

7. a) Spin the spinner five times to fill in the digits.

■.■■

0.■■

b) Add the two numbers.

c) Subtract the lesser number from the greater number.

d) Repeat nine times. What is the greatest value you calculated? What is the least value?

8. Lucas made punch to sell at a powwow. He combined 1.22 L of ginger ale, 0.76 L of orange juice, 0.89 L of grapefruit juice, and 0.56 L of raspberry juice. Then Lucas spilled some of the punch. He had 2.95 L left. How much did he spill?

9. Gabrielle is training for a race by running around the schoolyard twice. What might be the distance of the race that she is training for?

10. Why might you be able to solve $3 - 2.04$ by solving $3.00 - 2.04$? Why might you not?

Reading Strategy

Read the problem. In your own words, write what you are being asked to do.

32.62 m

16.32 m

schoolyard

31.71 m

20.35 m

3.3 Multiplying by Numbers Less than 1

YOU WILL NEED
- grid paper
- a calculator

Multiply by decimals less than 1.

LEARN ABOUT the Math

Yan has a picture that is 80 cm long by 60 cm wide. She is getting a piece of glass cut to cover the picture. She needs to calculate the area of the picture to figure out the price of the glass.

? What is the area of the picture in square metres?

A. Draw a model of a square metre on grid paper. Use a 10 × 10 array of 100 grid squares.

B. What fraction of a square metre does each grid square represent? Write the fraction as a decimal.

C. Represent Yan's picture on your model by colouring grid squares.

D. What is the length of the picture as a fraction of a metre? Write the fraction as a decimal.

E. What is the width of the picture as a fraction of a metre? Write the fraction as a decimal.

F. What expression represents the area of the picture?

G. What is the area of Yan's picture?

Reflecting

H. To determine the area of Yan's picture, you used a 10×10 grid to multiply tenths by tenths. Why does multiplying tenths by tenths always give an answer in hundredths?

I. How is the answer for 0.8×0.6 related to the answer for 8×6?

J. How do you know that when you multiply by a decimal less than 1, the product is less than you started with?
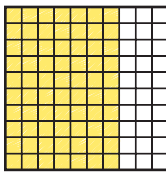
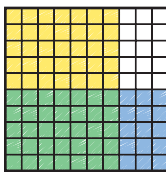# *WORK WITH* the Math

Example 1 | Multiplying using a grid

Calculate the area of a picture in a frame that is 70 cm long by 50 cm wide. Write the area in square metres.

Yan's Solution

Both the length and the width of the grid represent 1 m. I coloured 7 columns to show 0.7 of the grid.

I coloured 5 rows to represent 0.5 of the grid. The area where the blue 0.5 overlaps the yellow 0.7 represents 0.5 of 0.7.

There are 35 squares, so the area is 0.35 m^2.
The area of my frame is 0.35 m^2.

It also represents 0.5×0.7, since it is the area of a 0.5 by 0.7 rectangle. It is 0.35 of the whole grid.

The whole area of the grid represents 1 m^2.

Example 2 | Multiplying thousandths using a calculator

Julie dropped a rubber ball from a height of 8.2 m. Each time the ball bounced, it bounced to 0.355 times its previous height. About how high was the second bounce?

8.2 m
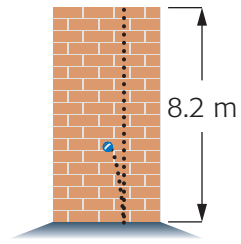
Julie's Solution

8 . 2 ⊠ 0 . 3 5 5 ⊟

2911

The first bounce was 2.911 m high.

To calculate the height of a bounce, I multiplied the height of the previous bounce by 0.355. I used a calculator.

I know that 0.355×8.2 should be close to 0.4×8.0. That's 3.2, so my calculation is reasonable.

2 . 9 1 1 ⊠ 0 . 3 5 5 ⊟

1033405

1.033 405 is about 1.0.
The second bounce was 1.0 m high.

I multiplied again by 0.355.

I expressed my answer to one decimal place, because this is the number of decimal places in the original height.

A Checking

1. Calculate using a 10×10 grid.
   a) 0.4×0.6 b) 0.2×0.7

2. Calculate, and then estimate to check if your answer is reasonable.
   a) What is the cost of 0.38 kg of birdseed at $0.95 for each kilogram?
   b) What is the cost of 0.56 kg of rolled oats at $0.88 for each kilogram?

B Practising

3. Calculate.
   a) 3.4×0.2 **b)** 7.6×0.8

4. Calculate.
   a) 0.2×0.9 **b)** 0.8×0.7

5. Predict the order of these six products from least to greatest. Check your prediction by calculating.
   a) 1.3×0.8 **c)** 1.5×0.2 **e)** 5.6×0.2
   b) 4.9×0.6 **d)** 10.6×0.3 **f)** 8.4×0.5

6. Place the digits 6, 7, and 8 so that the product is as close to 5 as possible: $0.\blacksquare \times \blacktriangle.\blacklozenge$.

7. In her backyard, Julie has a rabbit run that is 1.2 m long and 0.9 m wide. What is the area of the rabbit run?

8. Joseph Starblanket buys 1.89 kg of beads at $0.85 for each kilogram. Determine how much he pays. Use a calculator.

9. Dora's garden is 2.90 m long and 0.85 m wide. She decides to change her garden so that its length is 1 m less and its width is 1 m greater. What is the change in the area of her garden?

10. Why is it easier to multiply 0.64×0.5 mentally than it is to multiply 0.64×0.7 mentally?

11. Suppose that you multiply 2.34 by a decimal less than 1. What do you know about the answer?
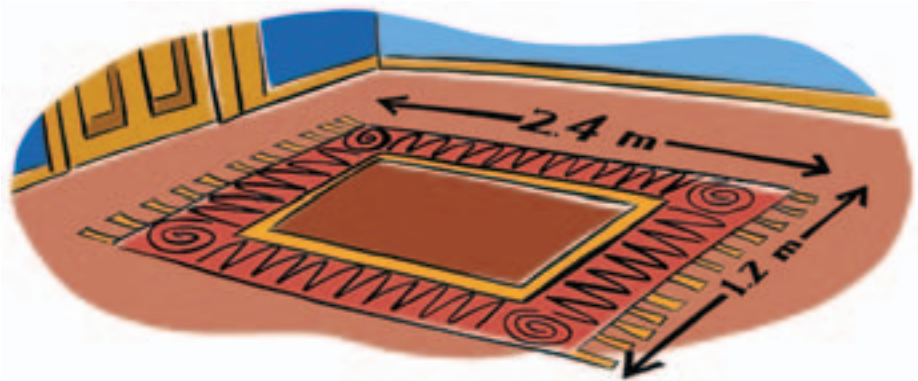
3.4

Multiplying by Numbers Greater than 1

GOAL

Multiply by decimals greater than 1.

LEARN ABOUT the Math

Max's mother is buying a rug to put in the entrance hall of their home. The area she wants to cover is 2.4 m long and 1.2 m wide.
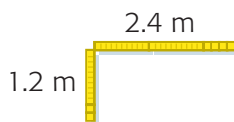

❓ **How big a rug does Max's mother need to cover the hall?**

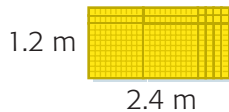**Example 1** | **Multiplying decimals using base ten blocks**

Determine the area of the rug that Max's mother needs.

Max's Solution

2.4 m
1.2 m

I decided to use base ten blocks to model a rectangle that is 2.4 m long and 1.2 m wide. A rod represents 1.0 m, so the length of a small cube represents 0.1 m. I put down rods and cubes to show the length and width of the rectangle.

1.2 m

2.4 m

To model the rug, I filled in the rectangle. Each flat represents $1.00 \text{ m} \times 1.00 \text{ m} = 1.00 \text{ m}^2$.
Since 10 rods make a flat, each rod represents 0.10 m^2.
Since 100 small cubes make a flat, each small cube represents 0.01 m^2.

1.2 m x 2.4 m = 2.88 m²
The area of the rug that we need is 2.9 m².

There are 2 flats, 8 rods, and 8 small cubes.
The area of the rug that we need is
$2.00 + 0.80 + 0.08 \text{ m}^2$.

Reflecting

A. Why is the product of two decimals greater than 1 always greater than both factors?

B. How is multiplying by a decimal greater than 1 the same as multiplying by a decimal less than 1?

WORK WITH *the Math*

Example 2 | Multiplying and dividing by 100

Julie uses 7.2 g of silver to make a small pin. Silver costs about $0.18 for each gram. Calculate the cost of the silver in the pin to the nearest cent.

Solution

0.18×7.2 is about $0.2 \times 7 = 1.4$

$0.18 \times 100 = 18$

$18 \times 7.2 = 129.6$

$129.6 \div 100 = 1.296$

The cost is $1.30.
This answer is reasonable because it is close to the estimate.

Estimate the amount in dollars.

Multiply 0.18 by 100 to get a whole number, so you can avoid having to multiply two decimals.

Multiply 18 by 7.2.

Divide by 100 to reverse the earlier multiplication by 100.

Example 3 | Placing a decimal point in a product

Max entered 6.42 × 13.5 on his calculator and got 8667.0. Is this result correct?

Solution

6.42 × 13.5 is about 6 × 10 = 60.

The decimal point could go in many places to get answers such as 0.8667, 8.667, 86.67, and 866.7. Estimating helps you determine the answer that is reasonable.

The product must be 86.67.

86.67 is closest to the estimate.

A Checking

1. Place the decimal point correctly in each product.
   a) 3.4 × 2 = 680   **b)** 26.50 × 2.2 = 5830

2. Estimate and then calculate.
   a) 4.5 × 3.6   **b)** 12.23 × 2.9

B Practising

3. Place the decimal point correctly in each product.
   a) 3.13 × 1.2 = 3756   **c)** 26.45 × 2.162 = 571849
   b) 15.45 × 3.2 = 4944   **d)** 321.06 × 11.3 = 3627978

4. Predict the order of these six products from greatest to least. Calculate to check your prediction.
   a) 32.25 × 1.8   **d)** 2.2 × 0.03
   b) 0.45 × 2.6   **e)** 3.67 × 1.01
   c) 12.347 × 0.64   **f)** 0.35 × 10.19

5. The decimal point is in the wrong place in each answer. Put it in the correct place, and explain the strategy you used.
   a) 45.66 × 12.2 = 5570.52   **c)** 1.09 × 30.65 = 334.085
   b) 0.78 × 1.023 = 79.794   **d)** 52.56 × 11.25 = 5913.0

6. Replace each blank with a number so that the products are in order from least to greatest.

■.4 × 5.■

■.8 × 4. 9

■.562 × ■.12■

7. What is the area of a garden patch that is 3.26 m wide and 5.13 m long?

8. Suppose Annik works 7.5 h each week. After working for a year, she gets a raise from $9.25 for each hour to $9.50 for each hour. How much more money will she earn in a week?

9. Louise wants to integrate the Franco-Albertan flag into the flag for La Conférence de la Francophonie. Her flag will be 2.41 m long and 1.7 m wide. Determine the area of the flag.

10. The adult height of a man is about 1.19 times his height at age 12. The adult height of a woman is about 1.07 times her height at age 12. Miguel is 1.5 m tall, and Romona is 1.6 m tall. Both are 12 years old. Predict how tall they will be as adults.

11. One postage stamp costs $1.85. How much does a book of 25 stamps cost?

12. Suppose that you have $3.00 and jellybeans cost $0.85 for each 100 g. Can you afford to buy 300 g of jellybeans?

13. A butcher sells ground beef for $2.25 for each kilogram. How much will 3.4 kg of ground beef cost?

14. Suki is painting her bedroom ceiling. The ceiling is 4.2 m long and 3.9 m wide. Suki has one can of paint, which will cover 12 m². Does she have enough paint to put two coats on the ceiling? Explain.

15. The Hendersons spend about $132.50 on food every week. There are 52 weeks in a year. How much do they spend on food every year?

16. Meagan says that, to multiply 1.3 × 1.3, you can multiply 1 × 1 and 0.3 × 0.3, and then add the products to get 1.09 for the answer. Do you agree? Explain.

Decimal Numbers **113**

3.5 Solve Problems Using Guessing and Testing

**GOAL**

Use guess and test to solve measurement problems.

LEARN ABOUT the Math

The students in Liam's class have drawn some pictures. The teacher puts the pictures in three groups. Group A pictures are 20.3 cm wide, group B pictures are 15.6 cm wide, and group C pictures are 8.4 cm wide. Liam is asked to pin up some of the pictures along the bottom of a bulletin board, from one end to the other, with no space between them.

185.6 cm

A   B   C

20.3 cm 15.6 cm 8.4 cm
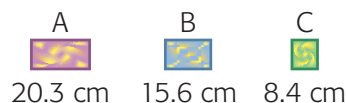
The bulletin board is 185.6 cm wide. Liam has to use about the same number of pictures from each group.

❓ How many pictures from each group should Liam use?

1 Understand the Problem

Liam knows that the bulletin board is 185.6 cm wide. He needs to use pictures from all three groups.

2 Make a Plan

Liam decides to try different combinations and see which is the right width.

A   B   C

3 Carry Out the Plan

Liam organizes the combinations he tries in a table.

A	B	C	Total width (cm)	Right width?
1	1	1	20.3 + 15.6 + 8.4 = 44.3	no, too short; try more
2	2	2	44.3 x 2 = 88.6	no, too short; try more
3	3	3	44.3 x 3 = 132.9	no, too short; try more
4	4	4	44.3 x 4 = 177.2	no, too short; try more
5	5	5	44.3 x 5 = 221.5	no, too long; try fewer
5	4	5	101.5 + 62.4 + 42.0 = 205.9	no, too long; try fewer
5	4	4	101.5 + 62.4 + 33.6 = 197.5	no, too long; try fewer
4	4	5	81.2 + 62.4 + 42.0 = 185.6	just right!

Liam can cover the width of the bulletin board with 4 pictures from group A, 4 from group B, and 5 from group C. Since he is using more pictures from group C, he decides to start with C and end with C. His pattern is C, A, B, C, A, B, C, A, B, C, A, B, C.

4 Look Back

Liam looks at his pattern and thinks it is correct since there are about the same number of drawings from each group. As well, his pattern fits the width of the bulletin board exactly. Liam estimates to check: 4×20.3 is about 80, 4×15.6 is about 60, and 5×8.4 is about 40. $80 + 60 + 40 = 180$, which is close to the width of the bulletin board.

Reflecting

A. How did the guess and test strategy help Liam solve the problem?

B. What other strategy could Liam have used to solve the problem?

WORK WITH the Math

Example | Solving a problem using guess and test

A rectangle has an area of 1.44 m². Neither the length nor the width is 1.00 m. What might the length and the width of the rectangle be?

Julie's Solution

① Understand the Problem

Julie knows that the area is 1.44 m². She also knows that the formula for the area of a rectangle is length multiplied by width. She knows that there is more than one possible answer, but she needs to determine only one answer.

② Make a Plan

Julie tries different combinations of the length and width to see which product gives the right area. She keeps track of her guesses in a table.

③ Carry Out the Plan

Length (m)	Width (m)	Area (m²)	Right area?
1.20	1.10	$1.20 \times 1.10 = 1.32$	too small, try longer width
1.20	1.30	$1.20 \times 1.30 = 1.56$	too big, try shorter width
1.20	1.20	$1.20 \times 1.20 = 1.44$	just right

④ Look Back

Julie checks her calculations. She notices that $12 \times 12 = 144$, so it makes sense that $1.2 \times 1.2 = 1.44$. She thinks she is correct.

A Checking

1. Liam had pictures that were 20.3 cm wide, 15.6 cm wide, and 8.4 cm wide. He posted all the pictures along one wall of the class. The pictures made a line that was 318.5 cm long. How many pictures of each width did Liam have?

■ cm

perimeter = 187.2 cm ▲ cm
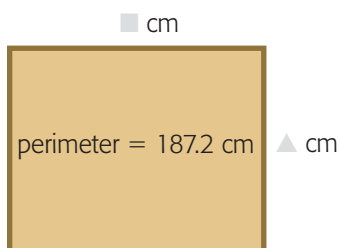
B Practising

2. Winnie is using the diagram at the left to make a wooden picture frame.
   a) List two different sets of dimensions (length and width) for the picture frame.
   b) Which set results in a greater area?

3. An Egyptian mummy is in a case that is 3 m high and has a volume of 24 m^3. The case is half as wide as it is long. What are the length and the width of the case?

4. Poul has 124.8 m of fencing to build a rectangular pen. He wants the pen to have the largest possible area. Calculate the length and width of the pen to one decimal place.

5. A small rectangular box has a width of 18 cm and a volume of 720 cm^3.
   a) Determine the height and the length of the box.
   b) Is there more than one answer? Explain.

6. The Leclaire family is choosing between two swimming pools for their backyard. Model A is almost square. Model B is about twice as long as it is wide. Both pools are 2.0 m deep and hold about 117.6 m^3 of water. Determine a possible length and width for each pool. Express your answers to the nearest tenth.

7. Jessica is helping her grandmother make a quilt with crocheted squares. The area of the material they have is 1.936 m^2. Her grandmother asked her to determine the number of squares they will need to crochet, and the length and width of each square in centimetres. What are two possible choices that Jessica can give her grandmother?

Decimal Numbers **117**

Mid-Chapter **Review**

Frequently Asked Questions

Q: **How can you add or subtract decimals?**

A: You can use base ten blocks and a place value mat. For example, this place value mat shows that $5.23 + 3.82 = 9.05$.

Tens	Ones	•	Tenths	Hundredths	Thousandths
+					
=					

Q: **How can you multiply two decimals?**
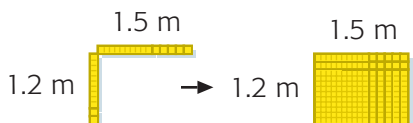
A1. You can use base ten blocks to represent the length and width of a rectangle. For example, to multiply 1.5 by 1.2, make a rectangle with 1.5 as the length and 1.2 as the width. Use 1 flat and 7 rods, as well as 10 small cubes, which are equal to one rod. So, $1.5 \times 1.2 = 1.8$.

1.5 m
1.2 m   1.5 m
1.2 m

A2: You can multiply related numbers and adjust the answer.

For example, 0.3×4.67 can be compared to 3×467.
$3 \times 467 = 1401$

You can estimate that 0.3×4.67 is about $0.5 \times 4 = 2$, so the answer must be 1.401.

Or, you can multiply 0.3 by 10 to get 3.
$3 \times 4.67 = 14.01$

Then divide by 10 to reverse the multiplication.
$14.01 \div 10 = 1.401$.

Practice

1. Predict the order of the four sums from least to greatest. Calculate to check your prediction.
   a) $2.67 + 6.24$
   c) $55.213 + 26.543 + 38.222$
   b) $65.331 + 21.951$
   d) $0.236 + 1.897$

2. Estimate each difference. What strategy did you use?
   a) $22.65 - 11.20$
   c) $258.688 - 23.126$
   b) $159.32 - 62.15$
   d) $30.265 - 6.697$

3. Anita and Joey have $95. They want to buy a guitar for $75.50, a package of new strings for $16.99, and a pick for $0.69. All prices include taxes.
   a) Estimate the total cost. Do Anita and Joey have enough money to buy these items?
   b) Calculate either the amount they are short or the amount they would have left over.

4. Calculate using a model.
   a) 0.2×0.9
   b) 0.6×0.4

5. Predict the order of the four products from greatest to least. Calculate to check your prediction.
   a) 0.9×3.28
   c) 1.4×5.3
   b) 6.93×0.33
   d) 6.3×2.1

6. In 2005, gas cost 79.9¢ for each litre. Hector put 40.5 L in his car. How much did he pay? Express your answer to the nearest cent.

7. This box holds 24 cubes of vegetable stock. Each cube has a side length of 0.8 cm. Determine the area of the bottom of the box.

8. A nickel is 0.185 cm thick. How thick is a roll of nickels worth $2?

9. Suppose that the mass of one egg is 0.065 kg and the mass of an empty egg carton is 16.85 g. One egg carton holds 12 eggs. What is the mass of six cartons of eggs?

0.8 cm
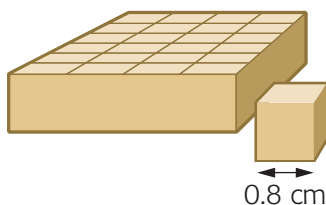
Dividing by Numbers Less than 1

YOU WILL NEED
- number lines
- pencil crayons
- a calculator

GOAL

Divide by one-digit decimals to solve problems.

LEARN ABOUT *the Math*

Fiona has a ribbon that is 0.84 m long. She is cutting the ribbon into pieces that are 0.07 m long.

How many pieces of ribbon will she have?

A. Estimate how many pieces Fiona can cut.

B. Draw a number line.

C. Show how many pieces of ribbon Fiona can cut using your number line.

Reflecting

D. Why can you solve the problem by dividing 0.84 by 0.07?

E. Why could you have solved 84 ÷ 7 instead?

WORK WITH *the Math*

Example 1 | Dividing decimals

Jean-Luc is pouring maple syrup from a pail into small jars to sell at a market. His pail contains 2.800 L of syrup. Each jar holds 0.350 L of syrup. How many jars can he fill?

Fiona's Solution

2800 ÷ 350 is about 3000 ÷ 300 = 10.
I estimate that he can fill 10 jars.
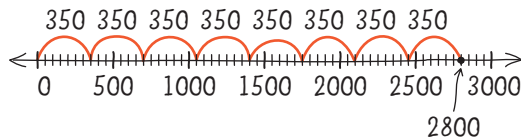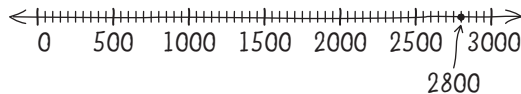
2.800 ÷ 0.350 = 8
There are 8 jumps, so Jean-Luc can fill 8 jars.

I estimated how many jars he can fill.
I wrote the amounts in millilitres to avoid dealing with decimals.
2.800 L = 2800 mL and 0.350 L = 350 mL

I drew a number line from 0 to 3000, with spaces of 50 because these numbers were convenient.

Each "jump" represents 1 jar.
I started at 0 and put a mark on the line for every jump of 350.

This answer is close to my estimate, so it is reasonable.

Example 2 | Dividing decimals using a calculator

Calculate 0.6 ÷ 0.125.

Matthew's Solution

0.125 is about 1 tenth, and there are about 6 tenths in 0.6. So 0.6 ÷ 0.125 is about 6.

0.6 ÷ 0.125 =

The answer is 4.8.

I estimated.

I divided using a calculator.

The answer is reasonable because it is close to my estimate.

Example 3 | Dividing using a grid

Suppose that Jean-Luc uses a 1.5 L pail of syrup to fill jars that hold 600 mL each. How many jars can he fill?

Yan's Solution
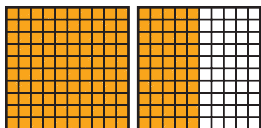
I represented 1.5 L with two 10 × 10 grids. Since 1 L equals 1000 mL, each little square represented 10 mL.
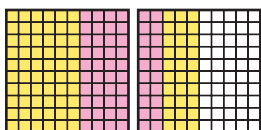
I coloured 150 squares in groups of 60 squares. Each group of 60 squares represented 600 mL, which fills one jar.

There are 2 groups of 60 squares and 30 squares left over. Since 30 squares are another half of a group, the amount left over is 0.5.

$1.500 \div 0.600 = 2.5$

Jean-Luc can fill 2.5 jars.

I knew that 0.6 + 0.6 = 1.2, so the answer is reasonable.

Example 4 | Dividing decimals using equivalents

Nolan has $1.75 to spend on pencils. Each pencil costs $0.05. How many pencils can he buy?

Nolan's Solution

$1.75 \div 0.05 = ?$

$1.75 \div 0.05$ is the same as $175 \div 5$.

$175 \div 5 = 35$
I can buy 35 pencils.

I needed to divide 1.75 by 0.05.

I knew that these are the same because the number of 5 hundredths in 175 hundredths is the same as the number of 5s in 175.

I divided.

A Checking

1. Calculate.
   a) $3.6 \div 0.18$ b) $7.8 \div 0.3$

2. Estimate each quotient.
   a) $3.2 \div 0.4$ b) $2.95 \div 0.5$

3. Calculate.
   a) $1.65 \div 0.3$ b) $4.59 \div 0.9$

B Practising

4. Calculate using mental math or a diagram.
   a) $3.6 \div 0.4$ b) $2.8 \div 0.4$ c) $2.5 \div 0.5$ d) $12.4 \div 0.3$

5. Calculate.
   a) $2.7 \div 0.45$ d) $14.8 \div 0.18$
   b) $3.1 \div 0.05$ e) $0.27 \div 0.04$
   c) $10.2 \div 0.14$ f) $1028.34 \div 0.45$

6. Xavier has 8.75 m of rope. He wants to divide it into equal pieces. How many pieces will there be if the pieces are each length below?
   a) 0.7 m   b) half a metre

7. How many 0.35 L glasses can you fill with a 1.65 L bottle of water?

8. Snails travel about 0.013 m each second. A football field is 100.06 m long. How long would a snail take to travel the length of a football field?

9. A small box of craft beads holds 0.44 kg of beads. How many small boxes could you fill with 5.06 kg of beads?

10. Suppose that one side of a die is 0.9 cm. How many dice could you fit in a box with a volume of 36.45 cm³? Remember that all the sides of a die have the same dimensions.

11. Why is the result for $1.25 \div 0.01$ the same as the result for 1.25×100?

12. Suppose that you divide 2.34 by a decimal less than 1. What do you know about the answer?

3.7 Dividing by Numbers Greater than 1
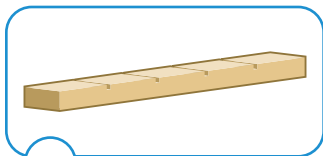
GOAL

Divide by decimals greater than 1.

LEARN ABOUT the Math

Max has a piece of lumber that is 2.4 m long. He must cut it into five equal pieces.

How long will each piece of lumber be?

A. Estimate how long each piece will be.

B. Figure out how long each piece will be, using a number line.

Reflecting

C. What strategies did you use to solve this problem?

D. Why is the quotient when you divide by 0.7 greater than if you divide by 1?

E. How did estimating help you check your answer to part B?

WORK WITH the Math

Example 1 | Dividing decimals using base ten blocks

Yan is making a workbench. For the top of the workbench, she plans to use a sheet of wood that has an area of 3.64 m² and a width of 1.4 m. How long will the top of the workbench be?

Yan's Solution
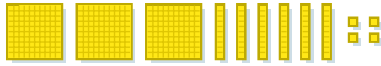
I used base ten blocks to represent the area.
The area is 3.64 m². I let a flat represent 1 whole.

I knew that 3 flats equal 3 wholes, 6 rods equal
6 tenths, and 4 small cubes equal 4 hundredths.

1.4 m

I needed to arrange the blocks to form a rectangle
with a width of 1.4.
I used 1 rod and 4 small cubes to represent the width.

2.6 m

1.4 m
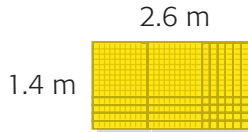
I arranged the other blocks to complete the rectangle.
I needed to regroup some of the blocks. 1 flat
became 10 rods, and 2 rods became 20 small cubes.
The length is equal to the area divided by the width:
$3.64 \div 1.4 = 2.6$.

The length of the rectangle was
2 flats and 6 rods, so the top of the
workbench will be 2.6 *m* long.

Example 2 | Dividing decimals using a calculator

The parking lot behind a family
restaurant is 91.8 m long. Each
parking space is 3.4 m wide. How
many cars does the lot hold?

car   car   car   car

3.4 m 3.4 m 3.4 m 3.4 m
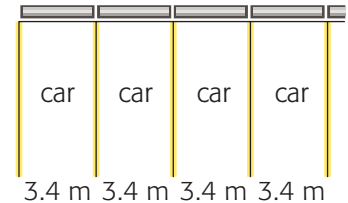
Liam's Solution

I needed to calculate $91.8 \div 3.4$.

The number of spaces is the same as the number of
cars the lot will hold. I needed to determine how many
3.4 m spaces are in 91.8 m. That's a division problem.

$91.8 \div 3.4$ is about $90 \div 3 = 30$.
The lot will hold about 30 cars.

I estimated first.

9 1 . 8 ÷ 3 . 4 =

27.

I calculated $91.8 \div 3.4$ on my calculator.
The answer is close to my estimate, so it is reasonable.

The lot will hold 27 cars.

Example 3 | Placing a decimal point in a quotient

Place the decimal point correctly in this quotient.
11.907 ÷ 2.1 = 567

Julie's Solution

Estimate the quotient.
11.907 ÷ 2.1 is about 12 ÷ 2 = 6.

The decimal point could go in many places. The answer is probably 0.567, 5.67, or 56.7, however. Estimating helps me decide which choice is reasonable.

The product must be 5.67.

5.67 is closer to the estimate of 6 than any other possible number.

A Checking

1. Place the decimal point correctly in each quotient.
   a) 5.7 ÷ 1.9 = 30
   b) 13.2 ÷ 2.4 = 55

2. Calculate. Describe which strategy you used.
   a) 2.369 ÷ 1.03
   b) 50.50 ÷ 5.05

3. Reece worked 13 h at the deli last week. His pay cheque for the week was $97.50. How much is he paid for each hour of work?

B Practising

4. Calculate.
   a) 3.6 ÷ 1.2
   b) 5.35 ÷ 2.14
   c) 2.25 ÷ 0.15
   d) 9.72 ÷ 2.7

5. Predict the order of the six quotients from least to greatest. Calculate to test your prediction.
   a) 8.4 ÷ 2.4
   b) 3.13 ÷ 3.13
   c) 10.2 ÷ 1.5
   d) 14.04 ÷ 3.12
   e) 89.688 ÷ 4.04
   f) 3286.976 ÷ 147.2

6. Estimate to check each answer. Correct each incorrect answer.
   a) 4.1 ÷ 2.4 = 1.75
   b) 6.72 ÷ 2.10 = 3.20
   c) 7.12 ÷ 1.20 = 60.0
   d) 1.48 ÷ 3.70 = 0.04

7. Kim has $11.50 in coins. How many coins does she have if all of the $11.50 is in each type of coin?

a) dimes   **b)** nickels   **c)** quarters   **d)** pennies

8. Nathan has a rope that is 11.4 m long. He wants to divide it into equal pieces. How many equal pieces will there be if the pieces are each length below?

a) 80 cm   **b)** 1.4 m   **c)** 0.7 m   **d)** half a metre

9. Suppose that milk is on sale for 87.5¢ for each litre. How many litres can you buy for $20?

10. Susan earned $191.25 last week. She was paid $8.50 for each hour. How many hours did she work?

11. There are 18 erasers in a box. The total mass of the box of erasers is 229.85 g. The empty box has a mass of 4.85 g. What is the mass of one eraser?

12. The adult height of a man is about 1.19 times his height at age 12. The adult height of a woman is about 1.07 times her height at age 12. Predict how tall each person was at age 12.
a) a man who is 1.8 m tall
b) a woman who is 1.8 m tall

13. Kyle is filling his brother's wading pool. The pool holds 180 L of water, and the hose supplies water at 22.5 L for each minute. How long will it take to fill the pool? Answer to the nearest minute.

14. The mass of a carton of six pizza pockets is 952.0 g. The mass of the empty carton is 72.0 g. Determine the mass of one pizza pocket. Express your answer to three decimal places.

15. Create three division problems that involve decimals. Exchange problems with a classmate, and write the solutions.

16. Suppose that you divide 2.34 by a decimal greater than 1. What do you know about the answer?

3.8 Using the Order of Operations with Decimals

GOAL

Evaluate expressions using the order of operations.

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order of operations

a set of rules people use when calculating to get the same answer:
Rule 1: Do the operations in brackets first.
Rule 2: Divide and multiply from left to right.
Rule 3: Add and subtract from left to right.
Hint: To remember these rules, think of **BDMAS**: **B**rackets, **D**ivide and **M**ultiply, **A**dd and **S**ubtract.

LEARN ABOUT the Math

Max and Fiona are putting wallpaper on one wall of a stage set. Fiona says, "To determine the area of wallpaper we need, we have to calculate $3.8 \times 2.8 - 1.2 \times 0.9$."

Max says, "So we need $3.8 \times 1.6 \times 0.9 = 5.5$ m^2 of wallpaper."

Fiona says, "I don't think that's right. We have to use the correct **order of operations**."

❓ What area of wallpaper do they need?
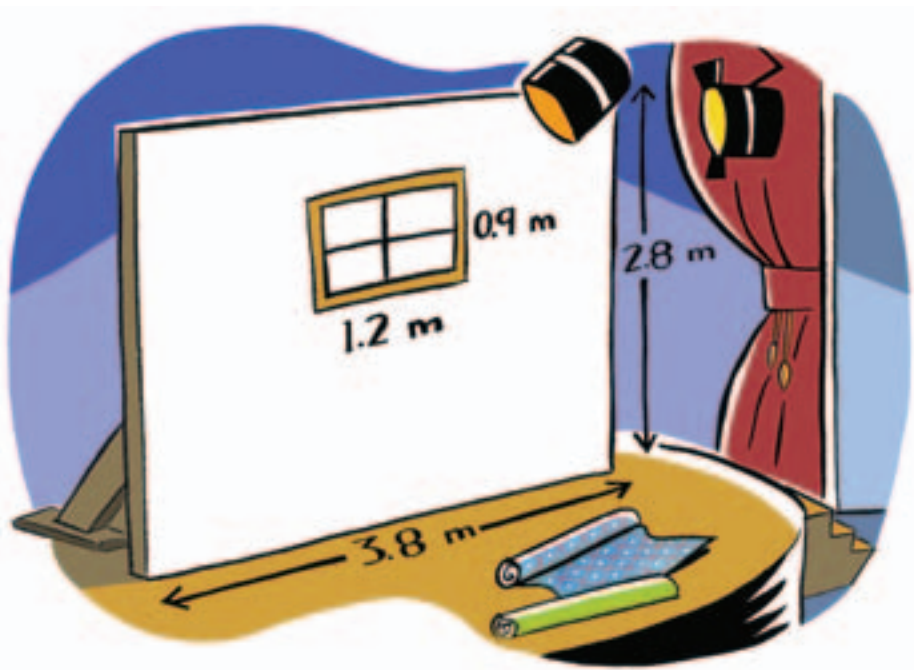
Example 1 | Calculating using order of operations

Calculate the area of wallpaper Max and Fiona need.

Fiona's Solution

Area of wallpaper
= area of wall – area of window
= (3.8 x 2.8) – (1.2 x 0.9)

Area = (3.8 x 2.8) – (1.2 x 0.9)
     = 10.64 – 1.08
     = 9.56
We need 9.56 m² of wallpaper.

The area of wallpaper we need is the area of the wall minus the area of the window.
I used brackets to separate the two parts.

I calculated the products in **B**rackets first. They were both **M**ultiplication. There was no **D**ivision or **A**ddition, so I **S**ubtracted from left to right next.

Reflecting

A. Explain why Max calculated the answer the way he did. Why was his calculation wrong?

B. Use the expression $3.2 + 1.5 \times 3.7$ to explain why it is important to have a set of rules for the order of operations that everyone uses.

WORK WITH the Math

Example 2 | Using the rules for the order of operations

Evaluate $32.28 - (1.2 \div 4 + 3.2)$.

Solution

$32.28 - (\underline{1.2 \div 4} + 3.2)$

$= 32.28 - (\underline{0.3 + 3.2})$

$= 32.28 - (3.5)$
$= 28.78$

Sometimes it helps to underline the operation you need to do in each step. Do the operations inside the brackets first. Divide, and then add.

Now do the operations outside the brackets.

Example 3 | Writing an expression with numbers

Write the following word expression as a numerical expression:
Multiply 4.1 by 6. Subtract 2. Divide by 4.

Solution

"Multiply 4.1 by 6" is 4.1×6.

Consider each part separately.

"Subtract 2."
$4.1 \times 6 - 2$

Just subtract 2.

"Divide by 4."
$(4.1 \times 6 - 2) \div 4$

Brackets are necessary. Without brackets, the expression would be "Multiply 4.1 by 6. Divide 2 by 4. Subtract the result from the product of 4.1 and 6." This is not the same.

A Checking

1. Calculate $15 - 12.6 \div 3 \times 2 - 1.5$ using the order of operations.

2. Which of the following expressions will give the same answer as the expression in question 1?
   A. $(15 - 12.6) \div 3 \times 2 - 1.5$ **C.** $15 - 12.6 \div (3 \times 2 - 1.5)$
   B. $15 - 12.6 \div 3 \times (2 - 1.5)$ **D.** $15 - (12.6 \div 3) \times 2 - 1.5$

3. Calculate.
   a) $(4.8 \times 4.8) - 15.02$ **b)** $7.12 - (4.8 - 2 \times 2)$

B Practising

4. Is each calculation correct? If not, correct it. Show your work.
   a) $12 \times 0.8 + 3 = 12.6$
   b) $12 + 0.8 \times 3 = 38.4$
   c) $5.6 \times 5.6 + 5 - 4.2 \div 2 = 16.08$
   d) $6.3 + 5 \times 5 - 4.2 \times 3 = 81.3$
   e) $4.2 \times 7 - 10.4 \div 2 + 2.9 \times 3 = 32.9$

5. Press these keys on your calculator:

3 . 8 ☒ 2 . 8 ⊟ 1 . 4 ☒ 0 . 9

Does your calculator follow the rules for order of operations? How do you know?

6.

Word expression	Numerical expression
Multiply 6.2 by 2, add 5.8 to this number, and divide the sum by 2.	6.2 × 2 + 5.8 ÷ 2

a) Explain how the word expression and the numerical expression are different.

b) How can you change the numerical expression to match the word expression?

c) Evaluate the word expression.

7. Evaluate. Show your work.

a) 3.5 × 10 + 10 × 3.2

b) 10.9 + (3.3 × 3.3 − 1.69) ÷ 3 − 5.2

c) 4.2 + 3.1 × 6.5 × (4.2 + 5.8)

d) 6.1 × (3.3 − 1.1 × 3) + 8 ÷ 8

8. Each solution has one error. Find the error and correct it. Explain what you did.

a) 4.2 × 4.2 − 5 × 3 = 17.64 − 5 × 3

        = 12.64 × 3

        = 37.92

b) 5.5 − 2.5 ÷ 0.5 + 3.1 × 2 + 6 = 3.0 ÷ 0.5 + 3.1 × 2 + 6

        = 6 + 3.1 × 2 + 6

        = 9.1 × 2 + 6

        = 18.2 + 6

        = 24.2

9. Write a numerical expression for each word expression.

a) Add 5.2 to 8.6. Multiply by 6.2.

b) Add 5.24 to 8.6. Multiply by 6. Subtract 5.2.

c) Divide 9.6 by 3.2. Multiply by 6.1. Subtract 8.5.

10. Explain why there are three rules for the order of operations, but there are five letters in BDMAS.

3.9 Expressing Fractions as Decimals

GOAL

Use division to express fractions as decimals.

LEARN ABOUT the Math

Julie and her friends are going on a hike. Julie buys 2 kg of trail mix to share.

🔍 How much trail mix will each hiker receive?

A. Suppose that there are three hikers. How can you use these linking cubes to show that each hiker will get $\frac{2}{3}$ of a kilogram of trail mix?

B. Write a division sentence to describe how each share can be calculated.

C. Calculate $2.000 \div 3$ using pencil and paper. What do you notice about the remainder after each step?

D. When you calculate $2 \div 3$ on your calculator, why does the display show 0.66666667 rather than the **repeating decimal** 0.6666 …, which is the actual answer?

E. Suppose that a different number of hikers go on the hike. Each hiker will receive an equal share of the 2 kg of trail mix. Complete the table on the next page. Write enough digits so that you can either see a pattern or see all the digits if the decimal is **terminating.**

> **repeating decimal**
>
> a decimal in which a block of one or more digits eventually repeats in a pattern; for example,
> $\frac{25}{99} = 0.252\ 525\ ...$,
> $\frac{31}{36} = 0.861\ 111\ ...$, and
> $\frac{1}{7} = 0.142\ 857\ 142\ 857$
> …. The dots mean that the numbers continue in the same pattern without stopping.

Number of hikers	Mass of trail mix each hiker receives (kg)	
	As a fraction	As a decimal
1	$\frac{2}{1}$	
2	$\frac{2}{2}$	
3	$\frac{2}{3}$	
4		
5		
6		
7		
8		
9		
10		

Communication | Tip

Some repeating decimals have large groups of repeating digits. This makes them awkward to write out. Instead of writing the repeating digits several times, we use a horizontal bar to mark them. This is called bar notation. For example, write 0.143 514 351 435 … as $0.\overline{1435}$, and write 0.999 … as $0.\overline{9}$.

Reflecting

F. Why can a terminating decimal always be written over a multiple of 10, such as $\frac{\blacksquare}{10}$, $\frac{\blacksquare}{100}$, or $\frac{\blacksquare}{1000}$?

G. Why can't $\frac{1}{3}$ be written in the form $\frac{\blacksquare}{10}$ or $\frac{\blacksquare}{100}$? Why can't it be a terminating decimal?

H. Why do you divide the numerator by the denominator to write a fraction as a decimal?

I. Why can a fraction such as $\frac{1}{8}$ or $\frac{1}{25}$ be written as an equivalent fraction with a denominator of 1000?

WORK WITH *the Math*

Example 1 | Comparing using equivalent decimals

Nolan has three bags of popcorn to share with seven friends. Fiona has four bags of popcorn to share with eight friends. The bags of popcorn are all the same size. Which group will receive larger portions?

Nolan's Solution

3 bags shared among 8 people is $\frac{3}{8}$.

4 bags shared among 9 people is $\frac{4}{9}$.

I included myself and Fiona in our groups. I calculated the size of each share in bags for both groups.

I knew that I could compare these numbers more easily by writing them as decimals.

$3 \div 8 = 0.375$ bags
$4 \div 9 = 0.\overline{4}$ bags

I used my calculator and divided the numerator in each fraction by its denominator.

$0.\overline{4} > 0.375$
Fiona's friends will receive the larger portions.

$0.\overline{4}$ is greater than 0.4, and 0.375 is less than 0.4.

Example 2 | Determining whether a decimal repeats

Determine whether the decimal equivalent of each fraction terminates or repeats. Order the fractions from least to greatest.

a) $\frac{2}{10}$ **b)** $\frac{7}{9}$ **c)** $\frac{8}{42}$ **d)** $\frac{53}{80}$

Max's Solution

If a decimal terminates, I will be able to express it as an equivalent fraction with a denominator that is a multiple of 10, such as 10, 100, or 1000.

a) $\frac{2}{10} = 0.2$; terminates

$\frac{2}{10}$ is already in this form, so I know that it terminates.

b) $\frac{7}{9} = 0.777 \ldots$; repeats

I know that $\frac{1}{9} = 0.111 \ldots$, so I know that $\frac{7}{9}$ must be 7 times as much.

c) $\frac{8}{42} = \frac{4}{21}$; repeats

Since 7 is a factor of 21, but 7 is not a factor of a multiple of 10, such as 1000 or 10 000, the decimal repeats.

I can rewrite $\frac{8}{42}$ as $\frac{4}{21}$. I tried to write $\frac{4}{21} = \frac{\blacksquare}{100}$, but there is no whole number I can multiply 21 by to get 100. I couldn't write the fraction as $\frac{\blacksquare}{1000}$ either.

d) $\frac{53}{80} = \frac{6625}{10\,000}$ or 0.6625; terminates

I tried writing $\frac{53}{80} = \frac{\blacksquare}{100}$, but this didn't work since $100 \div 80$ is not a whole number. Then I tried $\frac{53}{80} = \frac{\blacksquare}{1000}$. This didn't work either. I tried one more time. This worked because $80 = 8 \times 10$ and $10\,000 = 8 \times 125 \times 10$.

I checked my predictions with a calculator.

I was right!

a) $\frac{2}{10} = 0.2$ c) $\frac{8}{42} = 0.\overline{190\,476}$

b) $\frac{7}{9} = 0.\overline{7}$ d) $\frac{53}{80} = 0.6625$

$0.\overline{190\,476}, 0.2, 0.6625, 0.\overline{7}$

I ordered the decimals from least to greatest.

$\frac{8}{42}, \frac{2}{10}, \frac{53}{80}, \frac{7}{9}$

Then I ordered the fractions from least to greatest.

A Checking

1. Write each repeating decimal in bar notation.
   a) 0.555 555 555 …
   b) 0.134 561 345 613 456 …

2. Compare each pair of fractions using equivalent decimals. Replace each $\blacksquare$ with >, <, or =.
   a) $\frac{5}{16} \; \blacksquare \; \frac{2}{9}$
   b) $\frac{7}{11} \; \blacksquare \; \frac{5}{8}$
   c) $\frac{17}{20} \; \blacksquare \; \frac{11}{14}$

B Practising

3. Decide whether the decimal equivalent of each fraction terminates or repeats.

a) $\dfrac{3}{4}$ b) $\dfrac{5}{9}$ c) $\dfrac{9}{14}$ d) $\dfrac{19}{20}$

4. Write each decimal as a fraction.

a) 0.1625   b) 0.8550

5. If possible, write each fraction as a terminating decimal.

a) $\dfrac{14}{25}$ c) $\dfrac{1}{16}$ e) $\dfrac{19}{20}$

b) $\dfrac{5}{8}$ d) $\dfrac{4}{5}$ f) $\dfrac{22}{32}$

6. If possible, write each fraction as a repeating decimal.

a) $\dfrac{1}{6}$ c) $\dfrac{7}{11}$ e) $\dfrac{48}{49}$

b) $\dfrac{8}{9}$ d) $\dfrac{7}{15}$ f) $\dfrac{57}{111}$

7. Sort the fractions based on whether they are equivalent to a terminating decimal or a repeating decimal.

a) $\dfrac{4}{9}$ c) $\dfrac{5}{6}$ e) $\dfrac{5}{18}$

b) $\dfrac{3}{5}$ d) $\dfrac{15}{16}$ f) $\dfrac{19}{32}$

8. Order the fractions in question 7 from least to greatest.

9. a) Describe the following fraction pattern: $\dfrac{8}{9}, \dfrac{8}{99}, \dfrac{8}{999}, \ldots$. Write the next three fractions in the pattern.

     b) Rewrite the pattern using decimal equivalents for each of the six fractions.

     c) Describe the decimal pattern. Is the decimal pattern easier or harder to describe than the fraction pattern?

10. Express each fraction as a repeating decimal.

a) $\dfrac{1}{7}$ b) $\dfrac{2}{7}$ c) $\dfrac{3}{7}$

11. a) Describe a pattern in your answers for the previous question.

b) Predict the decimal equivalents of $\frac{4}{7}$ and $\frac{5}{7}$.

12. Replace each ■ with >, <, or =.

a) 0.2 ■ $0.\overline{2}$

b) $\frac{45}{99}$ ■ $0.\overline{45}$

c) $0.\overline{82}$ ■ $\frac{4}{5}$

d) $\frac{6}{11}$ ■ $\frac{7}{13}$

e) $0.357\ 357\ 357\ ...$ ■ $0.\overline{375}$

f) $\frac{2}{3}$ ■ 0.633

13. Order the numbers from least to greatest.

a) $\frac{1}{8}, \frac{5}{7}, 0.35, 0.\overline{39}, \frac{9}{10}$

b) $0.56, 0.5\overline{6}, 0.\overline{56}, \frac{5}{9}, \frac{27}{50}$

14. Predict the decimal equivalent of each fraction, using the fact that $\frac{1}{3} = 0.333....$

a) $\frac{2}{3}$

b) $\frac{1}{9}$

c) $\frac{1}{30}$

d) $\frac{4}{3}$

15. Calculate the decimal equivalent of each fraction.

a) $\frac{1}{12}$

b) $\frac{1}{28}$

c) $\frac{1}{44}$

d) $\frac{1}{52}$

16. Look at your answers for the previous question.

a) How are the decimal equivalents alike?

b) What do the denominators have in common?

17. The cost of a new toy is $1 after taxes. You and two friends want to split the cost evenly.

a) Express each person's share as a fraction.

b) Express the fraction as a decimal.

c) How much should each of you pay? Explain your decision.

d) Create a similar problem with a different fraction and solve it.

18. How can you tell, without calculating, that the decimal equivalent of $\frac{1}{33}$ repeats?

3.10 Expressing Decimals as Fractions

GOAL

Write decimals as fractions.

LEARN ABOUT the Math

Fiona says, "I know how to write a fraction as a decimal. I wonder how I can write a decimal as a fraction."

? How can you write a repeating decimal as a fraction?

A. Calculate each fraction as a repeating decimal: $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}$.

B. How could the decimal for $\frac{1}{9}$ have helped you predict the decimals for $\frac{2}{9}, \frac{3}{9}$, and $\frac{4}{9}$?

C. Calculate each fraction as a repeating decimal: $\frac{1}{99}, \frac{5}{99}, \frac{10}{99}, \frac{15}{99}$.

D. How could the decimal for $\frac{1}{99}$ have helped you predict the decimals for $\frac{5}{99}, \frac{10}{99}$, and $\frac{15}{99}$?

E. Predict the decimal equivalent for $\frac{1}{999}$.

F. How could you use what you have just learned to write $0.\overline{123}$ as a fraction?

Reflecting

G. Explain how to convert a repeating decimal to a fraction.

WORK WITH *the Math*

Example 1 | Expressing some decimals as fractions

Julie forgot her lunch. Yan offered Julie 0.35 of her sandwich.
Fiona offered Julie $\frac{1}{3}$ of her sandwich. Who offered more?

Yan's Solution

$$0.35 = \frac{35}{100}$$

Since 0.35 is 35 hundredths, I wrote 0.35 as $\frac{35}{100}$.

$$\overset{\times 3}{\underset{\times 3}{\frac{35}{100} = \frac{105}{300}}}$$

I wanted to compare the two numbers, so I wrote the fractions with the same denominator. I chose a denominator of 300. I multiplied the numerator and the denominator of $\frac{35}{100}$ by 3.

$$\overset{\times 100}{\underset{\times 100}{\frac{1}{3} = \frac{100}{300}}}$$

I wrote $\frac{1}{3}$ as an equivalent fraction with a denominator of 300. To do this, I multiplied both the numerator and the denominator by 100.

$$\frac{105}{300} > \frac{100}{300}$$

I compared the fractions.
105 is greater than 100, so $\frac{105}{300}$ is greater than $\frac{100}{300}$.

$0.35 > \frac{1}{3}$. I offered more.

Example 2 | Writing a repeating decimal as a fraction

Explain how you know that the fraction for $0.\overline{123}$ is greater than the fraction for 0.123.

Liam's Solution

$$0.\overline{123} = \frac{123}{999} \qquad 0.123 = \frac{123}{1000}$$

I wrote the fraction equivalent for each decimal.

$$\frac{123}{999} > \frac{123}{1000}$$

$\frac{1}{999}$ is greater than $\frac{1}{1000}$.

The fraction for $0.\overline{123}$ is greater than the fraction for 0.123.
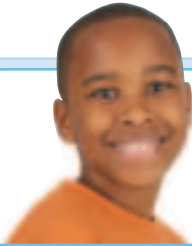
Example 3 | Writing a repeating decimal as a fraction

How do you know that $0.\overline{24} = \frac{24}{99}$?

Max's Solution

$0.\overline{01} \times 24 = 0.\overline{24}$

I know that $\frac{1}{99} = 0.\overline{01}$, so $\frac{24}{99}$ is 24 times as much.

I multiplied each $0.\overline{01}$ part by 24.

A Checking

1. Write each decimal as a fraction.
  a) 0.162
  b) 0.0777…
  c) 0.272 727…

2. Write each decimal as a fraction. Then replace each ■ with <, >, or =.
  a) $0.375 \ ■ \ \frac{1}{4}$
  b) $0.23 \ ■ \ \frac{1}{7}$
  c) $0.844 \ ■ \ \frac{22}{25}$

B Practising

3. Write each decimal as a fraction.
  a) $0.\overline{14}$
  b) $0.2\overline{73}$
  c) 0.0777…
  d) $4.1\overline{7}$
  e) 0.272 727…
  f) $0.\overline{767}$

4. Replace each ■ with <, >, or =.
  a) $0.416 \ ■ \ \frac{1}{4}$
  b) $0.52 \ ■ \ \frac{1}{2}$
  c) $0.\overline{6} \ ■ \ \frac{2}{3}$
  d) $0.6 \ ■ \ \frac{2}{3}$

5. Match each fraction with its decimal equivalent.
  A. $\frac{4}{7}$
  B. $\frac{7}{13}$
  C. $\frac{6}{11}$
  D. $\frac{2}{21}$
  a) $0.\overline{54}$
  b) $0.\overline{095\ 238}$
  c) $0.\overline{571\ 428}$
  d) $0.\overline{538\ 461}$

6. Explain how you know, without using a calculator, that $0.\overline{45}$ is greater than $\frac{9}{20}$.

7. Describe how to write a terminating decimal as a fraction. Give an example.

Race to 50

The goal of this game is to create and multiply decimals to get the greatest product.

Number of payers: 2 or more

How to Play

1. One player shuffles the cards and deals 3 cards to each player.

2. Players use two of their cards to create a decimal that is less than 1.

3. Players then multiply their decimal by the digit on the third card as a one-digit decimal. The digit in the hundredths place of the product is their score for the round.

4. The game continues, with players adding their score for each round to their score for the previous round. The first player to reach 50 points wins.

Yan's Turn

My cards were 7, 9, and 4.
I could multiply 0.49 x 0.7 = 0.343.
The answer is 0.343, so I'd get 4 points.
Or, I could multiply 0.97 x 0.4 = 0.388.
The answer is 0.388, so I'd get
8 points.
I'll choose the second way,
because this gives me
more points.

Repeating Decimal Patterns

Denis noticed that the decimal equivalent of $\frac{1}{13}$ is $0.\overline{076\ 923}$.
He put pairs of digits from the decimal into a table. He chose the second digit in the repeat for the y-value. Then he plotted the points, starting with (0, 7). Then he connected the points in the order he plotted them. He joined the last point with the first point he plotted.

Denis did this for all the fractions from $\frac{1}{13}$ to $\frac{12}{13}$. All his graphs were one of two shapes. Here are the two shapes:

x	y
0	7
7	6
6	9
9	2
2	3
3	0

Graph for $\frac{1}{13}, \frac{3}{13}, \frac{4}{13}, \frac{9}{13}, \frac{10}{13}$, and $\frac{12}{13}$

Graph for $\frac{2}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}$, and $\frac{11}{13}$
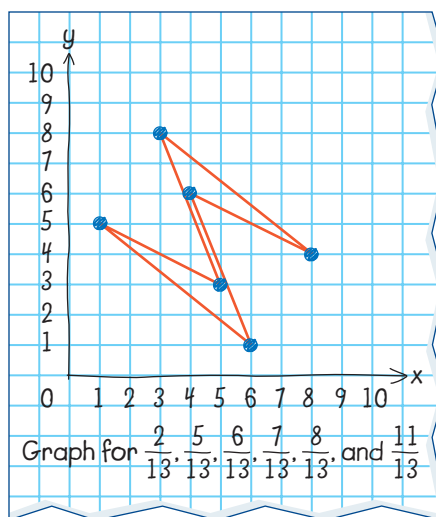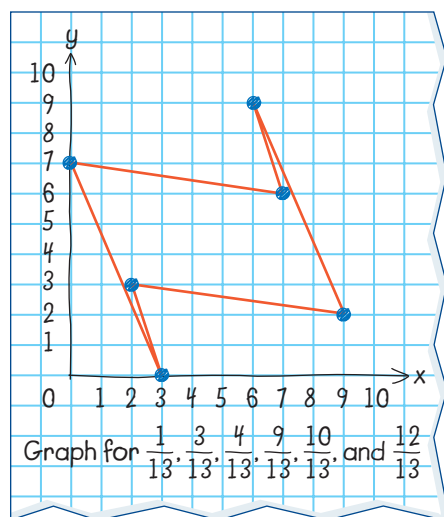
1. Choose one fraction family, such as $\frac{1}{7}, \frac{2}{7}, \dots, \frac{6}{7}$ or $\frac{1}{14}, \frac{2}{14}, \dots, \frac{13}{14}$. Write each fraction in the family as a repeating decimal.

2. Graph the pairs of digits in the repeating part of each decimal. Join the points in the order you plotted them.

3. How many different graphs did your fraction family have?

4. What shape of graph do you think you would get for a terminating decimal? (Hint: Recall that $0.375 = 0.375\ 000\dots$)

Chapter **Self-Test**

1. Calculate.
   a) $2.14 + 3.72 + 8.93$ c) $0.69 + 3.13$
   b) $2.032 + 6.352 + 9.986$ d) $765.43 + 21.2$

2. Replace each ▪ with >, <, or =.
   a) $29.35 - 22.65$ ▪ $44.65 - 39.35$
   b) $9.302 - 6.603$ ▪ $3.695 - 0.385$

3. Ethan has $2.65, Hannah has $4.25, and Sybil has $3.25. They need $4.95 to rent a video. How many videos can they rent?

4. Jason bought 0.7 kg of tomatoes. The tomatoes sell for $1.96 for each kilogram. How much did he pay?

5. Calculate.
   a) 1.38×2.7 c) 18.4×6.55
   b) 2.43×9.15 d) 69.207×41.5

6. Susan's mother filled her car's gas tank at the gas station. She bought 48.3 L, and her bill was $37.43. What was the price of the gas in cents for each litre?

7. Calculate.
   a) $6.4 \div 1.6$ b) $56.482 \div 6.4$

8. Predict whether the decimal equivalent of each fraction will terminate or repeat.
   a) $\dfrac{17}{20}$ b) $\dfrac{11}{16}$ c) $\dfrac{67}{99}$ d) $\dfrac{2}{7}$

9. Write each fraction in the previous question as a decimal.

10. Write each decimal as a fraction in lowest terms.
    a) 0.66 b) $1.\overline{3}$ c) 256.895 d) $73.\overline{25}$

What Do You Think Now?

Revisit What Do You Think? on page 99. How have your answers and explanations changed?

Chapter **Review**

Frequently Asked Questions

Q: **How do you divide a decimal by a decimal?**

A1: You can use a number line. For example, to divide 2.5 by 0.5, use a number line from 0 to 2.5. Count how many 0.5s are on the number line. There are five 0.5s, so 2.5 ÷ 0.5 = 5.
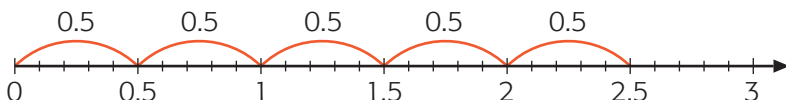
A2: You can use 10 × 10 grids. For example, to divide 2.5 by 0.4, colour three 10 × 10 grids to model 2.5. There are 6 sections that represent 0.4, and there are 10 squares left over. Since 10 squares are another fourth of a group, they represent 0.25. So 2.5 ÷ 0.4 = 6.25.

2.5   2.5 ÷ 0.4 = 6.25
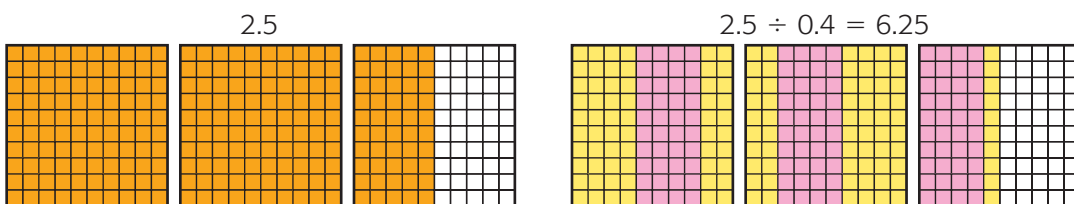
Q: **How can you write a fraction as a decimal?**

A: You can divide the numerator by the denominator. For example, the decimal equivalent of $\frac{1}{8}$ is 0.125.

1 ÷ 8 = 0.125

Q: **How can you write a decimal as a fraction?**

A: If the decimal terminates, use the decimal as the numerator and a number such as 10, 100, or 1000 as the denominator. For example, the fraction equivalent of 0.42 is $\frac{42}{100}$. If the decimal repeats, use the repeating part as the numerator and a number such as 9, 99, or 999 as the denominator. For example, the fraction equivalent of $0.\overline{42}$ is $\frac{42}{99}$.

Practice

1. Calculate.
   a) $3.5 + 370.6$
   b) $642.13 + 291.89$

2. Six students are holding hands to make the longest line they can. Their arm spans are 1.61 m, 1.66 m, 1.63 m, 1.72 m, 1.78 m, and 1.75 m. How long is their line?

3. Estimate, and then subtract.
   a) $39.8 - 12.6$
   b) $26.32 - 19.56$

4. Calculate.
   a) $57.68 - 39.39$
   b) $46.231 - 16.332$

5. An elevator can hold 650.0 kg. Three people moving a piano have masses of 80.5 kg, 72.8 kg, and 89.9 kg. The piano has a mass of 352.5 kg. Will the elevator hold the people and the piano?

6. Calculate each product. Explain which strategy you used.
   a) 9×0.8
   b) 215×0.6
   c) 57×0.4
   d) 0.6×0.9

7. Mariette's table is 0.8 m long and 0.4 m wide. Julie's table is 0.9 m long and 0.3 m wide.
   a) Estimate whose table has a bigger area. Explain your reasoning.
   b) Calculate the answer, and compare it with your estimate.

8. You want to multiply two decimals that are both less than 1. What do you know about the answer?

9. A single bus fare costs $2.35. A monthly pass costs $45.75.
   a) Céline estimates that she will ride the bus 25 times this month. Should she buy a monthly pass? Explain.
   b) Kamal estimates that he will ride the bus 18 times this month. Should he buy a monthly pass? Explain.

10. Karen mixed together three 1.36 L cartons of orange juice, two 0.59 L bottles of ginger ale, and 2.52 L of fruit juice. How much punch did Karen make?

11. The rungs of a rope ladder are 0.3 m apart. Rapunzel is climbing down the ladder from a window that is 3.69 m above ground. How many rungs will Rapunzel climb down?

12. You have a string that is 25.50 m long. You need to cut the string into 0.25 m pieces. Estimate how many pieces you can cut. How did you estimate?

Lesson 3.7

13. Predict the order of the four quotients from least to greatest. Calculate to check your prediction.

    a) $36.9 \div 3$ **c)** $0.6 \div 0.2$

    b) $121.5 \div 12.1$ **d)** $9.2 \div 3.1$

14. A stack of 50 sheets of paper is 0.85 cm high. How many sheets of paper are in a stack that is 48.45 cm high?

15. Estimate $1035.75 \div 24.5$. Check your estimate.

Lesson 3.8

16. Estimate, and then calculate.

    a) $(2.6 \times 6) + 25 - (3.2 \times 2.1)$

    b) $(6.9 \div 3) + (4.6 \times 0.23)$

Lesson 3.9

17. Write a fraction, in lowest terms, for each decimal.

    a) 0.8 **b)** 0.735 **c)** 0.92 **d)** 0.250

18. Does the decimal equivalent of each fraction terminate or repeat?

    a) $\dfrac{11}{25}$ **b)** $\dfrac{5}{6}$ **c)** $\dfrac{13}{15}$ **d)** $\dfrac{3}{8}$

19. Write each fraction in the previous question as a decimal.

20. Order the following numbers from least to greatest:

    $0.25, 0.2555 \ldots, 0.252\,525 \ldots, \dfrac{2}{9}, \dfrac{13}{15}$.

Lesson 3.10

21. Write the fraction equivalent of each decimal in lowest terms.

    a) 0.63 **b)** $0.\overline{63}$

22. Create a problem in which you multiply and divide decimals. Explain your solution.

Chapter **Task**

Planning a Party

You are organizing a party for a sports team. There are 10 players and 2 coaches. Each player has contributed $15.75. You have set aside money to buy each coach a $15 gift certificate from a sports store. You also need to buy
- food for the main course
- drinks
- desserts
- a souvenir for each player
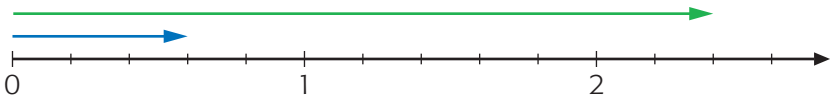
❓ What food and souvenirs will you buy?

A. Determine the total amount of money that you have to spend.

B. Check stores or advertising flyers for prices.

C. Make three different menus for the main course. You need to have enough food to serve four people with each menu. Determine how much money you can spend on the main course, drinks, and dessert, and still have money for souvenirs.

D. Prepare a report. Include the following information:
- the food, drinks, and desserts you will buy; the cost of each food item or drink; and the total cost
- the souvenirs you will buy, the cost of each souvenir, and the total cost

Cumulative **Review**

Note: Select ALL the correct answers to each question.

1. Which number is divisible by 10, 5, and 2?
   A. 1168   **B.** 2035   **C.** 21 890   **D.** 35 572

2. The number 637■16 is divisible by 4 and 8. What is ■?
   A. 2   **B.** 3   **C.** 5   **D.** 8

3. Which number is not a factor of 123 456?
   A. 2   **B.** 3   **C.** 6   **D.** 9

4. Which number is a factor of 1235?
   A. 2   **B.** 5   **C.** 4   **D.** 3

5. Which expression has a value that is between 0.5 and 1.5?
   A. $\dfrac{1}{4} + \dfrac{1}{5}$ **B.** $\dfrac{8}{3} + \dfrac{1}{9}$ **C.** $1\dfrac{5}{7} - \dfrac{5}{7}$ **D.** $4\dfrac{3}{4} - 2\dfrac{7}{8}$

6. Which pairs of fractions can be renamed as twelfths?
   A. $\dfrac{1}{2}, \dfrac{5}{6}$ **B.** $\dfrac{1}{3}, \dfrac{3}{4}$ **C.** $\dfrac{3}{7}, \dfrac{4}{5}$ **D.** $\dfrac{6}{7}, \dfrac{2}{3}$

7. Which expressions have a value that is a whole number?
   A. $2\dfrac{1}{3} + 4\dfrac{4}{6}$ **B.** $4\dfrac{1}{5} - 1\dfrac{6}{5}$ **C.** $6\dfrac{2}{7} - 3\dfrac{4}{7}$ **D.** $5\dfrac{1}{3} + 3\dfrac{8}{9}$

8. Which equation is represented on the following number line?

   A. $2\dfrac{2}{5} - \dfrac{3}{5} = 1\dfrac{4}{5}$ **C.** $\dfrac{4}{5} + 1\dfrac{3}{5} = 2\dfrac{2}{5}$

   B. $2\dfrac{2}{5} - 1\dfrac{3}{5} = \dfrac{4}{5}$ **D.** $2\dfrac{2}{5} + \dfrac{3}{5} = 3$

9. Which expressions have the same value as 0.5×750?
   A. $750 \div 2$ **B.** 5×750 **C.** 5×75 **D.** 0.05×75

10. Which expressions have the same value as 12.8×48?

A. 128×4.8 **C.** $2 \times 6.4 \times 4 \times 12$

B. 64×9.6 **D.** 25.6×24

11. Jake takes 0.8 mL of medicine each day. The bottle contains 16 mL of medicine. In how many days will he finish his medicine?

A. 2   **B.** 20   **C.** 8   **D.** 10

12. How much of 124.345 is 12.4345?

A. one tenth   **C.** one thousandth

B. one hundredth   **D.** one ten-thousandth

13. Estimate to determine which value is the product of 0.7×8.987.

A. 62.909   **C.** 0.629 09

B. 6.2909   **D.** 629.09

14. Which value is the best estimate for $12.078 \div 0.498\ 01$?

A. 24   **B.** 240   **C.** 2400   **D.** 24 000

15. Which expression has a product that is greater than 56.8?

A. 1.089×56.8 **B.** $0.000\ 123 \times 56.8$

16. Which expressions have a quotient of 12.3?

A. $369 \div 30$ **B.** $369 \div 3$ **C.** $36.9 \div 3$ **D.** $3.69 \div 0.3$

17. What is the value of ■ in $11.9 - 3 \times 6.4 + 79.2 =$ ■?

A. 136.16   **B.** 761.84   **C.** 71.9   **D.** 966.304

18. Which decimal represents the same amount as $\dfrac{1}{8}$?

A. 1.25   **B.** 0.125   **C.** 0.0125   **D.** 0.1025

19. Which fraction represents the same amount as 0.6?

A. $\dfrac{6}{100}$ **B.** $\dfrac{3}{5}$ **C.** $\dfrac{6}{1000}$ **D.** $\dfrac{30}{60}$

20. Which fraction is equivalent to a repeating decimal?

A. $\dfrac{1}{84}$ **B.** $\dfrac{1}{20}$ **C.** $\dfrac{1}{50}$ **D.** $\dfrac{1}{500}$